$$\mathbb{R} \text{ basis } {}^{n}\mathbb{C} \xleftarrow[\text{inj}]{\ulcorner} {}^{2n}\mathbb{R} \Leftrightarrow \begin{bmatrix}\ulcorner\\ \bar{\ulcorner}\end{bmatrix} \in {}^{2n}\mathbb{C}_{2n}^{\mathsf{C}} \text{ inv}$$

$$\Rightarrow: \quad \lrcorner \in {}^{2n}\mathbb{C}: \quad \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}\ulcorner\\ \bar{\ulcorner}\end{bmatrix} \lrcorner = \begin{bmatrix}\ulcorner\lrcorner\\ \bar{\ulcorner}\lrcorner\end{bmatrix} \Rightarrow \ulcorner\lrcorner = 0 = \bar{\ulcorner}\lrcorner$$

$$\Rightarrow \ulcorner\underbrace{\lrcorner + \bar{\lrcorner}} = \ulcorner\lrcorner + \ulcorner\bar{\lrcorner} = \ulcorner\lrcorner + \overline{\bar{\ulcorner}\lrcorner} = 0 \underset{\text{Vor}}{\Rightarrow} {}^{2n}\mathbb{R} \ni \lrcorner + \bar{\lrcorner} = 0$$

$$\ulcorner\underbrace{\lrcorner - \bar{\lrcorner}} = \ulcorner\lrcorner - \ulcorner\bar{\lrcorner} = \ulcorner\lrcorner - \overline{\bar{\ulcorner}\lrcorner} = 0 \Rightarrow \ulcorner i\underbrace{\lrcorner - \bar{\lrcorner}} = i\ulcorner\underbrace{\lrcorner - \bar{\lrcorner}} = 0 \underset{\text{Vor}}{\Rightarrow} {}^{2n}\mathbb{R} \ni i\underbrace{\lrcorner - \bar{\lrcorner}} = 0$$

$$\Rightarrow \lrcorner = \frac{\lrcorner + \bar{\lrcorner}}{2} + \frac{i\underbrace{\lrcorner - \bar{\lrcorner}}}{2i} = 0$$

$$\Leftarrow: \quad \lrcorner \in {}^{2n}\mathbb{R}: \quad \ulcorner\lrcorner = 0$$

$$\Rightarrow \bar{\ulcorner}\lrcorner = \bar{\ulcorner}\bar{\lrcorner} = \overline{\ulcorner\lrcorner} = 0 \Rightarrow \begin{bmatrix}\ulcorner\\ \bar{\ulcorner}\end{bmatrix} \lrcorner = \begin{bmatrix}\ulcorner\lrcorner\\ \bar{\ulcorner}\lrcorner\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \lrcorner = 0$$