$$\mathbf{A}\,{}^{2n}\mathbb{Z} = \mathbf{B}\,{}^{2n}\mathbb{Z} \Leftrightarrow \mathbf{A} = \mathbf{B}\mathbf{C}: \quad \mathbf{C} \in {}^{2n}\mathbb{Z}^{\mathsf{C}}_{2n}$$

$$\mathbf{A}\,{}^{2n}\mathbb{Z} \subset \mathbf{B}\,{}^{2n}\mathbb{Z} \Rightarrow \text{ cols } \mathbf{A}_q \in \mathbf{B}\,{}^{2n}\mathbb{Z} \Rightarrow \mathbf{A}_q = \mathbf{B}_p\,{}^{p}\mathbf{C}_q = \mathbf{B}\,\mathbf{C}_q$$

$${}^{p}\mathbf{C}_q \in \mathbb{Z}: \quad \mathbf{C}_q \in {}^{2n}\mathbb{Z} \Rightarrow \mathbf{A} = \mathbf{B}\mathbf{C}: \quad \mathbf{C} = \overbrace{\mathbf{C}_1 \cdot\cdot \mathbf{C}_{2n}} \in {}^{2n}\mathbb{Z}_{2n}$$

$$\text{analog } \mathbf{B} = \mathbf{A}\mathbf{D}: \quad \mathbf{D} \in {}^{2n}\mathbb{Z}_{2n}$$

$$\begin{bmatrix} \mathbf{A} \\ - \\ \mathbf{A} \end{bmatrix} \mathbf{D}\mathbf{C} = \begin{bmatrix} \mathbf{A}\mathbf{D} \\ - \\ \mathbf{A}\mathbf{D} \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathbf{A}\mathbf{D} \\ \hline \mathbf{A}\mathbf{D} \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathbf{B} \\ - \\ \mathbf{B} \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathbf{B}\mathbf{C} \\ - \\ \mathbf{B}\mathbf{C} \end{bmatrix} = \begin{bmatrix} \mathbf{B}\mathbf{C} \\ \hline \mathbf{B}\mathbf{C} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ - \\ \mathbf{A} \end{bmatrix}$$

$$\underset{\text{inv}}{\Rightarrow} \mathbf{D}\mathbf{C} = {}^{2n}I_{2n} = \mathbf{C}\mathbf{D} \Rightarrow \mathbf{C} \in {}^{2n}\mathbb{Z}^{\mathsf{C}}_{2n}$$