

$$\text{Wahrheitswerte} \begin{cases} 0 = \varnothing & \text{falsch} \\ 1 = (0) & \text{wahr} \end{cases}$$

$$2 = 0|1 \text{ Boole alg}$$

$$\bar{\alpha} = 1 - \alpha$$

$$\alpha \# \beta = \begin{cases} \alpha \wedge \beta = \alpha \beta \\ \alpha \vee \beta = \alpha \vee \beta \\ \alpha \rightarrow \beta = \underline{1 - \alpha} \vee \beta \end{cases}$$

$$\text{W-Funktion } \mathcal{P} \xrightarrow{\tau} 2 = 0|1 \quad : \quad \tau \in 2^{\mathcal{P}}$$

$$\bigwedge \tau \in 2^{\mathcal{P}} \bigwedge A \in \bar{\mathcal{P}} \bigvee_{\text{char}}^{\text{eind}} \hat{\tau} A = \hat{A} \tau \in 2 \begin{cases} \hat{P} \stackrel{\text{Ext}}{=} \tau P \\ \hat{\neg} A = 1 - \hat{A} \\ \hat{A \# B} = \hat{A} \# \hat{B} \end{cases}$$

$$2 \ni \begin{cases} \hat{\tau}(A \wedge B) = \hat{\tau}(A) \hat{\tau}(B) \\ \hat{\tau}(A \vee B) = \hat{\tau}(A) \vee \hat{\tau}(B) \\ \hat{\tau}(A \rightarrow B) = (1 - \hat{\tau}(A)) \vee \hat{\tau}(B) \end{cases}$$

$$k \in 2 \ni \begin{cases} \hat{P} = \tau P \\ \hat{\neg} \begin{bmatrix} A & k \end{bmatrix} = 1 - k \\ \hat{\#} \begin{bmatrix} A_1 & k_1 \end{bmatrix} \begin{bmatrix} A_2 & k_2 \end{bmatrix} = \underline{k_1} \# \underline{k_2} \end{cases}$$

$$\stackrel{\text{Rek}}{\text{Satz}} \bigvee_{*} \hat{\tau} \underbrace{f_n A_1 \cdots A_n}_{*} = \hat{f}_n \underbrace{A_1 | \hat{\tau} A_1 \cdots A_n | \hat{\tau} A_n}_{*}$$

$$\begin{cases} \hat{\tau} P = \hat{P} = \tau P \\ \hat{\tau} \bar{A} = \hat{\neg} \begin{bmatrix} A & \hat{\tau} A \end{bmatrix} = 1 - \hat{\tau} A \\ \hat{\tau} \underline{A \# B} = \hat{\#} \begin{bmatrix} A & \hat{\tau} A \end{bmatrix} \begin{bmatrix} B & \hat{\tau} B \end{bmatrix} = \underline{\hat{\tau} A} \# \underline{\hat{\tau} B} \end{cases}$$

$$2^{\mathcal{P}} \xrightarrow[\text{char}]{\hat{A}} 2: \quad \hat{A} \tau = \hat{\tau} A \text{ char function} \begin{cases} \hat{P} \tau = \tau P \\ \hat{\neg} A \tau = 1 - \hat{A} \tau \\ \hat{A \# B} \tau = \underline{\hat{A} \tau} \# \underline{\hat{B} \tau} \end{cases}$$

$$T \in \bar{\mathcal{P}} \text{ taut} \Leftrightarrow \hat{T} = 1 \Leftrightarrow \bigwedge_{\tau \in 2^{\mathcal{P}}} \hat{\tau} T = 1 \text{ immer wahr}$$

$$A \in \bar{\mathcal{P}} \text{ erfüllbar} \Leftrightarrow \hat{A} \neq 0 \Leftrightarrow \bigvee_{\tau \in 2^{\mathcal{P}}} \hat{\tau} A = 1 \text{ manchmal wahr}$$

$$F \in \bar{\mathcal{P}} \text{ absurd} \Leftrightarrow \hat{F} = 0 \Leftrightarrow \bigwedge_{\tau \in 2^{\mathcal{P}}} \hat{\tau} F = 0 \text{ nie wahr}$$

$$T = \underline{A \rightarrow B} \text{ taut} \Leftrightarrow \hat{A} \leq \hat{B} \Leftrightarrow \bigwedge_{\tau \in 2^{\mathcal{P}}} \hat{\tau} A \leq \hat{\tau} B$$

$$\underline{1 - \hat{\tau} A} \vee \hat{\tau} B = \hat{\tau} \underline{A \rightarrow B} = 1 \Leftrightarrow \underline{\hat{\tau} A = 1 \Rightarrow \hat{\tau} B = 1} \Leftrightarrow \hat{\tau} A \leq \hat{\tau} B$$

$$\text{Exportation taut } \underline{A \wedge B} \rightarrow C \leftrightarrow A \rightarrow \underline{B \rightarrow C}$$

$\hat{A}$	11110000
$\hat{B}$	11001100
$\hat{C}$	10101010
$1 - \hat{A}\hat{B}$	00111111
$1 - \hat{A}$	00001111
$1 - \hat{B}$	00110011
$\underline{1 - \hat{B}} \vee \hat{C}$	10111011
$\underline{1 - \hat{A}\hat{B}} \vee \hat{C} = \underline{1 - \hat{A}} \vee \underline{1 - \hat{B}} \vee \hat{C}$	10111111

finite List of 17 tautologies

$$A \rightarrow A$$

$$A \rightarrow \underline{B \rightarrow A}$$

$$\underline{A \wedge \bar{A}} \rightarrow B \text{ ex falso quodlibet : } B \rightarrow \underline{A \vee \bar{A}} \text{ ex quolibet verum}$$

$$\underline{A \rightarrow B} \rightarrow \underline{\bar{B} \rightarrow \bar{A}} \text{ Kontraposition}$$

$$A \rightarrow \underline{A \vee B} : A \rightarrow \underline{B \vee A}$$

$$\underline{A \wedge B} \rightarrow A : \underline{A \wedge B} \rightarrow B$$

$$\underline{A \rightarrow B} \rightarrow \underline{\underline{B \rightarrow C} \rightarrow \underline{A \rightarrow C}} : \underline{\underline{A \rightarrow B \rightarrow C} \rightarrow \underline{A \rightarrow C} \rightarrow \underline{B \rightarrow C}}$$

$$\underline{\underline{A \rightarrow C} \rightarrow \underline{B \rightarrow C} \rightarrow \underline{A \vee B} \rightarrow \underline{C}} : \underline{\underline{C \rightarrow A} \rightarrow \underline{C \rightarrow B} \rightarrow \underline{C \rightarrow A \wedge B}}$$

$$\underline{\underline{A \wedge B} \vee C} \leftrightarrow \underline{\underline{A \vee C} \wedge \underline{B \vee C}} : \underline{\underline{A \vee B} \wedge C} \leftrightarrow \underline{\underline{A \wedge C} \vee \underline{B \wedge C}} \text{ distributiv}$$