

Formeln $A:B:C \in \bar{\mathcal{P}}$ mit Regeln

$$0\text{-stellig } \bar{\mathcal{P}} = \left\{ \begin{array}{l} A \xrightarrow{a} A: \quad A \xrightarrow{b} B \rightarrow A: \quad \underline{A \rightarrow B} \xrightarrow{c} \bar{B} \rightarrow \bar{A}: \quad : \underline{A \wedge \bar{A}} \xrightarrow{d} B: \quad B \xrightarrow{5} \underline{A \vee \bar{A}} \\ A \rightarrow \underline{A \vee B} \leftarrow B: \quad A \leftarrow \underline{A \wedge B} \rightarrow B \\ \left\{ \begin{array}{l} \underline{A \rightarrow B} \xrightarrow{6} \underline{B \rightarrow C \rightarrow A \rightarrow C}: \quad \underline{A \rightarrow B \rightarrow C} \xrightarrow{6} \underline{A \rightarrow C \rightarrow B \rightarrow C} \\ \underline{A \rightarrow C} \xrightarrow{7} \underline{B \rightarrow C \rightarrow \underline{A \vee B} \rightarrow C}: \quad \underline{C \rightarrow A} \xrightarrow{7} \underline{C \rightarrow B \rightarrow C \rightarrow \underline{A \wedge B}} \end{array} \right. \\ \underline{A \wedge B} \vee C \xleftrightarrow{\text{dist}} \underline{A \vee C} \wedge \underline{B \vee C}: \quad \underline{A \vee B} \wedge C \xleftrightarrow{\text{dist}} \underline{A \wedge C} \vee \underline{B \wedge C} \end{array} \right.$$

$$2\text{-stellig } R = \frac{\overline{A \rightarrow B} | A | B}{A:B \in \bar{\mathcal{P}}} \text{ modus ponens}$$

$$\text{ableitbar } \bar{\bar{\mathcal{P}}} = \overline{\bar{\mathcal{P}}}$$

$$A \vee \bar{A} \in \bar{\bar{\mathcal{P}}}$$

$$A \xrightarrow[1]{0} A: \quad \underline{A \rightarrow A} \xrightarrow[\underline{A \vee \bar{A}}]{0} : \quad A \vee \bar{A}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \right\} \Rightarrow A \rightarrow C$$

$$A \xrightarrow[\text{Vor}]{\rightarrow} B: \quad B \xrightarrow[\text{Vor}]{\rightarrow} C: \quad \underline{A \rightarrow B} \xrightarrow{0} \underline{B \rightarrow C \rightarrow A \rightarrow C}: \quad \underline{B \rightarrow C} \xrightarrow{2} \underline{A \rightarrow C}: \quad A \xrightarrow{2} C$$

$$\left\{ \begin{array}{l} \underline{A \wedge B} \rightarrow \underline{B \wedge A} \\ \underline{A \vee B} \rightarrow \underline{B \vee A} \end{array} \right.$$

$$\begin{array}{l} \underline{A \wedge B} \xrightarrow{0} A: \quad \underline{A \wedge B} \xrightarrow{0} B: \quad \underline{A \wedge B} \rightarrow \bar{B} \xrightarrow{0} \overline{\underline{A \wedge B} \rightarrow A \rightarrow \underline{A \wedge B} \rightarrow \underline{B \wedge A}} \\ : \quad \underline{A \wedge B} \rightarrow A \xrightarrow{2} \underline{A \wedge B} \rightarrow \underline{B \wedge A}: \quad \underline{A \wedge B} \xrightarrow{2} \underline{B \wedge A} \\ A \xrightarrow{0} \underline{B \vee A}: \quad B \xrightarrow{0} \underline{B \vee A}: \quad \underline{A \rightarrow B \vee A} \xrightarrow{0} \underline{B \rightarrow B \vee A} \rightarrow \underline{A \vee B \rightarrow B \vee A} \\ : \quad \underline{B \rightarrow B \vee A} \rightarrow \underline{A \vee B \rightarrow B \vee A}: \quad \underline{A \vee B} \rightarrow \underline{B \vee A} \end{array}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \end{array} \right\} \Rightarrow A \rightarrow \underline{B \wedge C}$$

$$\left\{ \begin{array}{l} A \rightarrow C \\ B \rightarrow C \end{array} \right\} \Rightarrow \underline{A \vee B} \rightarrow C$$

$$A \xrightarrow{\text{Vor}} B: A \xrightarrow{\text{Vor}} C: \overline{A \rightarrow B} \xrightarrow{0} \overline{A \rightarrow C \rightarrow A \rightarrow \overline{B \wedge C}}: \underline{A \rightarrow C} \xrightarrow{2} \underline{A \rightarrow \overline{B \wedge C}}: A \xrightarrow{2} \overline{B \wedge C}$$

$$A \xrightarrow{\text{Vor}} C: B \xrightarrow{\text{Vor}} C: \underline{A \rightarrow C} \xrightarrow{0} \overline{B \rightarrow C \rightarrow \underline{A \vee B} \rightarrow C}: \overline{B \rightarrow C} \xrightarrow{2} \overline{\underline{A \vee B} \rightarrow C}: \underline{A \vee B} \xrightarrow{2} C$$

$$\left\{ \begin{array}{l} B \rightarrow \underline{A \vee \bar{A}} \wedge B \\ \underline{A \wedge \bar{A}} \vee B \rightarrow B \end{array} \right.$$

$$B \xrightarrow{0} \underline{A \vee \bar{A}}: B \xrightarrow{0} B: B \xrightarrow{\text{Lem}} \overline{A \vee \bar{A}} \wedge B: \underline{A \wedge \bar{A}} \xrightarrow{0} B: B \xrightarrow{0} B: \overline{A \wedge \bar{A}} \vee B \xrightarrow{\text{Lem}} B$$

$$\left\{ \begin{array}{l} A \rightarrow \bar{\bar{A}} \\ \bar{\bar{A}} \rightarrow A \end{array} \right.$$

$$A \xrightarrow{0} \underline{\bar{A} \vee \bar{\bar{A}}}: A \xrightarrow{0} A: A \xrightarrow{\text{Lem}} \overline{\bar{A} \vee \bar{\bar{A}}} \wedge A: \overline{\bar{A} \vee \bar{\bar{A}}} \wedge A \xrightarrow{0} \overline{\bar{A} \wedge A} \vee \overline{\bar{\bar{A}} \wedge A}: A \xrightarrow{\text{Lem}} \overline{\bar{A} \wedge A} \vee \overline{\bar{\bar{A}} \wedge A}$$

$$: \overline{\bar{A} \wedge A} \vee \overline{\bar{\bar{A}} \wedge A} \xrightarrow{\text{Lem}} \bar{\bar{A}} \wedge A: A \xrightarrow{\text{Lem}} \bar{\bar{A}} \wedge A: \underline{\bar{\bar{A}} \wedge A} \xrightarrow{0} \bar{\bar{A}}: A \xrightarrow{\text{Lem}} \bar{\bar{A}}$$

$$\bar{\bar{A}} \xrightarrow{0} \underline{\bar{A} \vee A}: \bar{\bar{A}} \xrightarrow{0} \bar{\bar{A}}: \bar{\bar{A}} \xrightarrow{\text{Lem}} \overline{\bar{A} \vee A} \wedge \bar{\bar{A}}: \overline{\bar{A} \vee A} \wedge \bar{\bar{A}} \xrightarrow{0} \overline{\bar{A} \wedge \bar{\bar{A}}} \vee \overline{A \wedge \bar{\bar{A}}}: \bar{\bar{A}} \xrightarrow{\text{Lem}} \overline{\bar{A} \wedge \bar{\bar{A}}} \vee \overline{A \wedge \bar{\bar{A}}}$$

$$: \overline{\bar{A} \wedge \bar{\bar{A}}} \vee \overline{A \wedge \bar{\bar{A}}} \xrightarrow{\text{Lem}} A \wedge \bar{\bar{A}}: \bar{\bar{A}} \xrightarrow{\text{Lem}} A \wedge \bar{\bar{A}}: \underline{A \wedge \bar{\bar{A}}} \xrightarrow{0} A: \bar{\bar{A}} \xrightarrow{\text{Lem}} A$$