$$|A|^{\gamma}_{x} = \frac{y \in |A|}{x \neq y \neq \gamma y}: \quad |\gamma|^{x}_{A} = \bigcup_{y \in |A|^{\gamma}_{x}} |\gamma y| = \bigcup_{|A| \ni y \neq x}^{\gamma y \neq y} |\gamma y| \subset X: \quad |A| \cup |\gamma|^{x}_{A} \text{ finit}$$

$$x^{A}_{\gamma} = \begin{cases} x & x \notin |\gamma|^{x}_{A} \\ \min X \llcorner |A| \cup |\gamma|^{x}_{A} & x \in |\gamma|^{x}_{A} \end{cases} \Longrightarrow x^{A}_{\gamma} \notin |\gamma|^{x}_{A}: \quad X = \{ x_0 : x_1 : \cdots \}$$

$$z = x^{A}_{\gamma}: \quad \alpha \underset{z}{\sim} \beta \Longrightarrow \underbrace{\check{\alpha} \bigcirc \gamma}_{x}{}^{\beta z} \hat{A} = \underbrace{\check{\beta} \bigcirc \gamma^{z}_{x}} \hat{A}$$

$$\bigwedge y \in |A|: \quad \underbrace{\check{\alpha} \bigcirc \gamma}_{x}{}^{\beta z} y \underset{\text{zz}}{\equiv} \underbrace{\check{\beta} \bigcirc \gamma^{z}_{x}} y = \check{\beta} \underbrace{\gamma^{z}_{x} y} \underset{\text{Trg}}{\Longrightarrow}$$

$$y = x: \quad \underbrace{\check{\alpha} \bigcirc \gamma}_{x}{}^{\beta z} x = \beta z = \check{\beta} \underbrace{\gamma^{z}_{x} x}_{= z}: \quad y \neq a: \quad \underbrace{\check{\alpha} \bigcirc \gamma}_{x}{}^{\beta z} y = \underbrace{\check{\alpha} \bigcirc \gamma} y = \check{\alpha} \gamma y \underset{\text{zz}}{\equiv} \check{\beta} \gamma y = \check{\beta} \underbrace{\gamma^{z}_{x} y}_{= \gamma y}$$

$$\gamma y = y \Longrightarrow z \neq y \Longrightarrow \check{\alpha} \gamma y = \alpha y \underset{z \neq y}{=} \beta y = \check{\beta} \gamma y$$

$$z = x \Longrightarrow z = x \neq y: \quad z \neq a \underset{\text{Def}}{\Longrightarrow} z \notin |A| \ni y \Longrightarrow z \neq y$$

$$\gamma y \neq y \Longrightarrow z \notin |\gamma y| \Longrightarrow \alpha \underset{|\gamma y|}{=} \beta \underset{\text{trg}}{\Longrightarrow} \check{\alpha} \gamma y = \check{\beta} \gamma y$$

$$y \in |A| \llcorner x: \quad \gamma y \neq y: \quad z = x \overset{\text{Def}}{\Longrightarrow} z = x \notin |\gamma y|: \quad z \neq a \underset{\text{Def}}{\Longrightarrow} z \notin |\gamma y|$$

$$\bigwedge \gamma \in \overline{O \cup X}^X \bigwedge A \in \overline{\mathcal{P}\overline{O \cup X}} \bigvee_{\text{Subst}}^{\text{eind}} \gamma \bigcirc A \in \overline{\mathcal{P}\overline{O \cup X}} \begin{cases} \gamma \bigcirc \underbrace{\neg A} = \neg \overbrace{\gamma \bigcirc A} \\ \gamma \bigcirc \underbrace{A \sharp B} = \underbrace{\gamma \bigcirc A} \sharp \underbrace{\gamma \bigcirc B} \\ \gamma \bigcirc \underbrace{p_m t_1 \cdots t_m} = p_m \underbrace{\check{\gamma} t_1} \cdots \underbrace{\check{\gamma} t_m} \\ \gamma \bigcirc \overbrace{\sharp_x A} = \sharp_{x^A_\gamma} \underbrace{\gamma^{x^A_\gamma}_x \bigcirc A} \end{cases}$$

$$\gamma \in \overline{O \cup X}^X \xrightarrow{\phi} \overline{\mathcal{P}\overline{O \cup X}} \ni \phi\gamma \begin{cases} \overbrace{p_m t_1 \cdots t_m}^{\sim} \gamma = p_m \underbrace{\check{\gamma} t_1} \cdots \underbrace{\check{\gamma} t_m} \\ \tilde{\sharp}_x \underbrace{A|\phi} \gamma = \sharp_{x^A_\gamma} \phi \gamma^{x^A_\gamma}_x \\ \tilde{\neg} \underbrace{A|\phi} \gamma = \neg \underbrace{\phi\gamma} \\ \tilde{\sharp} \underbrace{A_1|\phi_1 : A_2|\phi_2} \gamma = \underbrace{\phi_1 \gamma} \sharp \underbrace{\phi_2 \gamma} \end{cases}$$

$$\bigwedge_\alpha \bigwedge_\gamma \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} A \underset{*}{=} \hat{\alpha} \underbrace{\gamma \bigcirc A}$$

$$\text{Ind } \mathcal{P}\overline{O \cup X} \subset \frac{A \in \overline{\mathcal{P}\overline{O \cup X}}}{*} \underset{\text{abg}}{\subset} \overline{\mathcal{P}\overline{O \cup X}}$$

$$\begin{cases} \hat{\alpha} \overbrace{\gamma \bigcirc \underbrace{p_m t_1 \cdots t_m}} = \hat{\alpha} \overbrace{p_m \underbrace{\check{\gamma} t_1} \cdots \underbrace{\check{\gamma} t_m}} = \hat{P}_m \overbrace{\check{\alpha} \underbrace{\check{\gamma} t_1} \cdots \check{\alpha} \underbrace{\check{\gamma} t_m}} \overset{*}{=} \hat{P}_m \underbrace{\overbrace{\check{\alpha} \bigcirc \gamma}^{\vee} t_1 \cdots \overbrace{\check{\alpha} \bigcirc \gamma}^{\vee} t_m} = \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} \underbrace{p_m t_1 \cdots t_m} \\ \hat{\alpha} \overbrace{\gamma \bigcirc \underbrace{\neg A}} = \hat{\alpha} \overbrace{\neg \underbrace{\gamma \bigcirc A}} = 1 - \hat{\alpha} \underbrace{\gamma \bigcirc A} \underset{\text{Ind}}{=} 1 - \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} A = \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} \underbrace{\neg A} \\ \hat{\alpha} \overbrace{\gamma \bigcirc \underbrace{A \sharp B}} = \hat{\alpha} \overbrace{\underbrace{\gamma \bigcirc A} \sharp \underbrace{\gamma \bigcirc B}} = \overbrace{\hat{\alpha} \underbrace{\gamma \bigcirc A}} \sharp \overbrace{\hat{\alpha} \underbrace{\gamma \bigcirc B}} \underset{\text{Ind}}{=} \underbrace{\overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} A} \sharp \underbrace{\overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} B} = \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} \underbrace{A \sharp B} \\ \hat{\alpha} \underbrace{\gamma \bigcirc \overbrace{\sharp_x A}} = \hat{\alpha} \overbrace{\sharp_z \underbrace{\gamma^z_x \bigcirc A}} = \sharp_u \underbrace{\hat{\alpha}^u_z \overbrace{\gamma^z_x \bigcirc A}} \underset{\text{Ind}}{=} \sharp_u \underbrace{\overbrace{\check{\alpha}^u_z \bigcirc \gamma^z_x}^{\wedge} A} \underset{\text{trg}}{\overset{**}{=}} \sharp_u \underbrace{\overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge}}_x{}^u A = \overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge} \overbrace{\sharp_x A} \end{cases}$$

$$\text{da } \overbrace{\check{\alpha} \bigcirc \gamma}^{\vee} \overset{*}{=} \check{\alpha} \bigcirc \check{\gamma} \text{ und } \alpha \underset{z}{\sim} \alpha^u_z: \quad \alpha^u_z z = u \underset{\text{trg}}{\Longrightarrow} \underbrace{\overbrace{\check{\alpha} \bigcirc \gamma}^{\wedge}}_x{}^u A \overset{**}{=} \overbrace{\check{\alpha}^u_z \bigcirc \gamma^z_x}^{\wedge} A$$

$$\operatorname{rg}\gamma \bigcirc A \underset{*}{=} \operatorname{rg} A$$

$$\operatorname{Ind} \mathcal{P}\overline{O \cup X} \subset \frac{A \in \overline{\mathcal{P}\overline{O \cup X}}}{*} \underset{\text{abg}}{\subset} \overline{\mathcal{P}\overline{O \cup X}}$$

$$\begin{cases} \operatorname{rg}\gamma \bigcirc \underbrace{p_m t_1 \cdots t_m} = \operatorname{rg} p_m \underbrace{\overset{\vee}{\gamma} t_1} \cdots \underbrace{\overset{\vee}{\gamma} t_m} = 0 = \operatorname{rg} p_m t_1 \cdots t_m \\ \operatorname{rg}\gamma \bigcirc \underbrace{\neg A} = \operatorname{rg} \neg \overbrace{\gamma \bigcirc A} = 1 + \operatorname{rg} \overbrace{\gamma \bigcirc A} \underset{\text{Ind}}{=} 1 + \operatorname{rg} A \underset{\text{Rek}}{=} \operatorname{rg} \neg A \\ \operatorname{rg}\gamma \bigcirc \overbrace{\underset{x}{\sharp} A} = \operatorname{rg} \underset{x_\gamma^A}{\sharp} \underbrace{\gamma_x^{x_\gamma^A} \bigcirc A} = 1 + \operatorname{rg} \underbrace{\gamma_x^{x_\gamma^A} \bigcirc A} \underset{\text{Ind}}{=} 1 + \operatorname{rg} A = \operatorname{rg} \underset{x}{\sharp} A \\ \operatorname{rg}\gamma \bigcirc \underbrace{A \sharp B} = \operatorname{rg} \underbrace{\gamma \bigcirc A} \sharp \underbrace{\gamma \bigcirc B} = 1 + \operatorname{rg} \underbrace{\gamma \bigcirc A} + \operatorname{rg} \underbrace{\gamma \bigcirc B} \underset{\text{Ind}}{=} 1 + \operatorname{rg} A + \operatorname{rg} B = \operatorname{rg} A \sharp B \end{cases}$$

$$\iota_{x_0|x_2}^{x_4|s_2 x_1 x_1} \bigcirc \bigvee_{x_0} \overset{2}{p}\, x_0\, s_2\, x_1\, x_2 = \bigvee_{x_0} p\, x_0\, s_2\, x_1\, s_2\, x_1\, x_1$$

$$A = p\, x_0\, s_2\, x_1\, x_2 : \quad \gamma = \iota_{x_0|x_2}^{x_4|s_2 x_1 x_1} : \quad |A| = \{ x_0 : x_1 : x_2 \} : \quad \gamma\, x_1 = x_1 : \quad \gamma\, x_2 = s_2\, x_1\, x_1 \neq a_2$$

$$\Longrightarrow |A|_{x_0}^{\gamma} = \frac{y \in |A|}{x_0 \neq y \neq \gamma y} = (x_2) : \quad |\gamma\, x_2| = |s_2\, x_1\, x_1| = (x_1) \Longrightarrow x_0 \notin |\gamma\, x_2| \Longrightarrow z \underset{\text{Def}}{=} x_0 : \quad \gamma_{x_0}^z\, x_0 = z = x_0$$

$$\gamma_{x_0}^z\, x_1 = \gamma\, x_1 = x_1 : \quad \gamma_{x_0}^z\, x_2 = \gamma\, x_2 = s_2\, x_1\, x_1 \Longrightarrow \gamma_{x_0}^z = \iota_{x_2}^{s_2 x_1 x_1}$$

$$\Longrightarrow \gamma \bigcirc \bigvee_{x_0} A = \bigvee_{z} \gamma_{x_0}^z \bigcirc A = \bigvee_{x_0} \iota_{x_2}^{s_2 x_1 x_1} \bigcirc \underbrace{p x_0 s_2 x_1 x_2} = \bigvee_{x_0} p\, x_0\, s_2\, x_1\, s_2\, x_1\, x_1$$

$$\iota^{x_4|x_0}_{x_0|x_1} \bigwedge_{x_0} p\, x_0\, s_2\, x_1\, x_2 = \bigwedge_{x_3} \overset{2}{p}\, x_3\, s_2\, x_0\, x_2$$

$$A = p\, x_0\, s_2\, x_1\, x_2 : \quad \gamma = \iota^{x_4|x_0}_{x_0|x_1} : \quad |A| = \{x_0 : x_1 : x_2\} : \quad \gamma\, x_1 = x_0 \neq a_1 : \quad \gamma\, x_2 = x_2 : \quad |A|^{\gamma}_{x_0} = \frac{y \in |A|}{x_0 \neq y \ni \gamma y} = (x_1)$$

$$x_0 \in |\gamma\, x_1| = (x_0) \underset{\text{Def}}{\Longrightarrow} z = \min\ \left(X \llcorner |A| \cup |\gamma x_1|\right) = \min\ \left(X \llcorner x_0 : x_1 : x_2\right) = x_3$$

$$\gamma^{x_3}_{x_0}\, x_0 = x_3 : \quad \gamma^{x_3}_{x_0}\, x_1 = \gamma\, x_1 = x_0 : \quad \gamma^{x_3}_{x_0}\, x_2 = \gamma\, x_2 = x_2 \Longrightarrow \gamma^{x_3}_{x_0} = \iota^{x_3|x_0}_{x_0|x_1}$$

$$\gamma \bigcirc \bigwedge_{x_0} p\, x_0\, s_2\, x_1\, x_2 = \bigwedge_{x_3} \gamma^{x_3}_{x_0} \bigcirc \underbrace{p x_0 s_2 x_1 x_2} = \bigwedge_{x_3} \iota^{x_3|x_0}_{x_0|x_1} \bigcirc \underbrace{p x_0 s_2 x_1 x_2} = \bigwedge_{x_3} p\, x_3\, s_2\, x_0\, x_2$$