

$$\mathcal{S} = \mathcal{S}_{\pm} \xrightarrow[\text{spin}]{} M^e \xleftarrow[\text{VB}]{} E$$

$$M_{\triangleleft_{\infty}} \mathcal{S}_{-} \xleftarrow[\text{Dir}]{D} M_{\triangleleft_{\infty}} \mathcal{S}_{+}$$

$$M_{\triangleleft_{\infty}}^2 \mathcal{S} \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \\ \hline \overset{*}{D} & 0 \end{array}} M_{\triangleleft_{\infty}}^2 \mathcal{S}$$

$$\mathcal{S} \mathbf{z} E = \mathcal{S}_{\pm} \mathbf{z} E \rightarrow M^e$$

$$M_{\triangleleft_{\infty}} \mathcal{S}_{-} \mathbf{z} E \xleftarrow[\text{Dir}]{D \mathbf{z} \iota_E} M_{\triangleleft_{\infty}} \mathcal{S}_{+} \mathbf{z} E$$

$$M_{\triangleleft_{\infty}} \mathcal{S} \mathbf{z} E \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \mathbf{z} \iota_E \\ \hline \overset{*}{D} \mathbf{z} \iota_E & 0 \end{array}} M_{\triangleleft_{\infty}} \mathcal{S} \mathbf{z} E$$

$$D = V \widehat{\overset{*}{D} D}^{1/2}$$

$$\text{even K-cycle } [D] = {}^{M^e}\triangleleft_{\sigma}\mathbb{C} \ltimes {}^{M^2}\triangleleft_{\infty}\mathcal{S}_{\pm} \mathbf{z} E | V \mathbf{z} \iota_E \in K_0 \underbrace{{}^{M^e}\triangleleft_{\sigma}\mathbb{C}} = K^0 \left(M^e \right)$$