

$$\mathcal{S} = \mathcal{S}_{\pm} \xrightarrow{\text{spin}} N \xleftarrow{\text{hol VB}} E$$

$${}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}_- \xleftarrow[\text{Dir}]{D} {}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}_+$$

$${}^{N^2}\mathop{\triangleleft}\limits_{\infty}\mathcal{S} \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \\ \hline \overset{*}{D} & 0 \end{array}} {}^{N^2}\mathop{\triangleleft}\limits_{\infty}\mathcal{S}$$

$$\mathcal{S}\mathbf{x}E = \mathcal{S}_{\pm}\mathbf{x}E \rightarrow N$$

$${}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}_-\mathbf{x}E \xleftarrow[\text{Dir}]{D\mathbf{x}\iota_E} {}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}_+\mathbf{x}E$$

$${}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}\mathbf{x}E \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D\mathbf{x}\iota_E \\ \hline \overset{*}{D}\mathbf{x}\iota_E & 0 \end{array}} {}^N\mathop{\triangleleft}\limits_{\infty}\mathcal{S}\mathbf{x}E$$

$$D = V \widehat{\overset{*}{D}D}^{1/2}$$

$$\text{even K-cycle } [D] = {}^N\mathop{\triangleleft}\limits_{\circ}\mathbb{C} \ltimes {}^{N^2}\mathop{\triangleleft}\limits_{\infty}\mathcal{S}_{\pm}\mathbf{x}E|V\mathbf{x}\iota_E \in K_0(\underbrace{{}^N\mathop{\triangleleft}\limits_{\circ}\mathbb{C}}) = K^0(N)$$