

$$\mathcal{S} = \mathcal{S}_{\pm} \xrightarrow[\text{spin}]{} \overset{\dagger}{T}M \xleftarrow[\text{VB}]{} E$$

$$\overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S}_{-} \xleftarrow[\text{Dir}]{D} \overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S}_{+}$$

$$\overset{\dagger}{T}M \mathop{\triangleleft}_{\infty}^2 \mathcal{S} \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \\ \hline \overset{*}{D} & 0 \end{array}} \overset{\dagger}{T}M \mathop{\triangleleft}_{\infty}^2 \mathcal{S}$$

$$\mathcal{S} \mathbf{x} E = \mathcal{S}_{\pm} \mathbf{x} E \rightarrow \overset{\dagger}{T}M$$

$$\overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S}_{-} \mathbf{x} E \xleftarrow[\text{Dir}]{D \mathbf{x} \iota_E} \overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S}_{+} \mathbf{x} E$$

$$\overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S} \mathbf{x} E \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \mathbf{x} \iota_E \\ \hline \overset{*}{D} \mathbf{x} \iota_E & 0 \end{array}} \overset{\dagger}{T}M \mathop{\triangleleft}_{\infty} \mathcal{S} \mathbf{x} E$$

$$D = V \widehat{\overset{*}{D} D}^{1/2}$$

$$\text{even K-cycle } [D] = \overset{\dagger}{T}M \mathop{\triangleleft}_{\circ} \mathbb{C} \ltimes \overset{\dagger}{T}M \mathop{\triangleleft}_{\infty}^2 \mathcal{S}_{\pm} \mathbf{x} E | V \mathbf{x} \iota_E \in K_0 \underbrace{\overset{\dagger}{T}M \mathop{\triangleleft}_{\circ} \mathbb{C}} = K^0 \left(\overset{\dagger}{T}M \right)$$