

$$\mathcal{S} = \mathcal{S}_{\pm} \xrightarrow{\text{spin}} M \times \overset{+}{B} \xleftarrow{\text{hol VB}} E$$

$$M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S}_{-} \xleftarrow[\text{Dir}]{D} M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S}_{+}$$

$$M \times \overset{+}{B} \underset{\infty}{\triangleleft}^2 \mathcal{S} \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \\ \hline \overset{*}{D} & 0 \end{array}} M \times \overset{+}{B} \underset{\infty}{\triangleleft}^2 \mathcal{S}$$

$$\mathcal{S} \mathbf{z} E = \mathcal{S}_{\pm} \mathbf{z} E \rightarrow M \times \overset{+}{B}$$

$$M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S}_{-} \mathbf{z} E \xleftarrow[\text{Dir}]{D \mathbf{z} \iota_E} M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S}_{+} \mathbf{z} E$$

$$M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S} \mathbf{z} E \xleftarrow[\text{Dir}]{\begin{array}{c|c} 0 & D \mathbf{z} \iota_E \\ \hline \overset{*}{D} \mathbf{z} \iota_E & 0 \end{array}} M \times \overset{+}{B} \underset{\infty}{\triangleleft} \mathcal{S} \mathbf{z} E$$

$$D = V \widehat{\overset{*}{D} D}^{1/2}$$

$$\text{even K-cycle } [D] = M \times \overset{+}{B} \underset{o}{\triangleleft} \mathbb{C} \ltimes M \times \overset{+}{B} \underset{\infty}{\triangleleft}^2 \mathcal{S}_{\pm} \mathbf{z} E | V \mathbf{z} \iota_E \in K_0 \underbrace{M \times \overset{+}{B} \underset{o}{\triangleleft} \mathbb{C}} = K^0 \left(M \times \overset{+}{B} \right)$$