Poincare series

$$\underset{\bullet}{\psi} = \sum_{g \in \Gamma/\Gamma_\psi} \psi \bigcirc g^{-1}$$

$$\Pi \xrightarrow{\psi} \mathbb{C}$$

$$\Gamma_\psi = \frac{g \in \Gamma}{\psi \bigcirc g = \psi}$$

$$\Gamma = \mathbb{Z}_2^{\complement} \cap N = \begin{array}{c|c} 1 & \mathbb{Z} \\ \hline 0 & 1 \end{array}$$

$\Re\nu < -1$: $k \in \mathbb{Z}$: $N-$ inv function

$$^{z}\psi_\nu = \sqrt{\mathcal{I}z}^{1-\nu} \Longrightarrow \underset{\bullet}{\psi}_\nu = E_{(1-\nu)/2} \text{ Eisenstein}$$

$$^{z}\psi_\nu^k = \sqrt{\mathcal{I}z}^{1-\nu} \exp(2\pi ikz) \Longrightarrow \underset{\bullet}{\psi}_\nu^k = E_{(1-\nu)/2}^k \text{ Selberg}$$

$$A = \begin{array}{c|c} a^{1/2} & 0 \\ \hline 0 & a^{-1/2} \end{array}$$

$$\Gamma_\psi = \pm \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}$$

A homo function

$$^{az}\psi = \sqrt{a}^{\varrho - 1}$$

$$\nu \text{ eigenvalue } \mathcal{E} = \mathcal{E}^*: \mathbb{R}^2 \xrightarrow[m]{2} \mathbb{C}$$

$$\varrho \text{ eigenvalue } \mathcal{F} = x\,\partial_x - \xi\,\partial_\xi = -\mathcal{F}^*: \mathbb{R}^2 \xrightarrow[m]{2} \mathbb{C}$$

$$\text{auto Eisen } x|\xi \mapsto \overline{\xi}^{\nu-1} \underset{\text{Radon}}{\Leftrightarrow} E_{(1-\nu)/2} \text{ modu Eisen}$$

$$\text{auto bi-Eisen } x|\xi \mapsto \overline{x}^{\frac{\varrho+\nu}{2}-1}\,\overline{\xi}^{\frac{\varrho-\nu}{2}} \underset{\text{Radon}}{\Leftrightarrow} \mathcal{P}_{\frac{\nu-1}{2}}\left(-i\frac{\Re z}{\mathcal{I}z}\right) + \mathcal{P}_{\frac{\nu-1}{2}}\left(i\frac{\Re z}{\mathcal{I}z}\right) = {}^{z}\psi: \quad \varrho = 1$$

$$\text{auto bi-Eisen } x|\xi \mapsto \overline{x}^{\frac{\varrho+\nu}{2}-1}\,\overline{\xi}^{\frac{\varrho-\nu}{2}} \underset{\text{Radon}}{\Leftrightarrow} = {}^{z}\psi_{\varrho:\nu} + \psi_{\varrho:-\nu}: \quad 0 < \Re\varrho < 2: \quad \psi_{\varrho:\nu}(z) = \sqrt{\mathcal{I}z}^{\varrho-1} \chi_{\varrho:\nu}\left(\frac{\Re z}{\mathcal{I}z}\right)$$

$$\text{divergent } \underset{\bullet}{\psi} = \frac{1}{2}\sum_{g \in \Gamma} \psi(gz)$$

$$-1-\nu \text{ distributionon } \mathbb{R}^2 \Leftrightarrow \frac{1-\nu^2}{4} \text{ eigenfunction } \Delta \text{ on } \Pi$$

$$\underset{\bullet}{\psi}_{\varrho:\nu}$$