

$$\begin{aligned}
x_t &= f_{t-1}^{v_t} \frac{u_{t-1}^{v_{t-1}}}{\iota} f_{t-2}^{v_{t-1}} \frac{u_{t-2}^{v_{t-2}}}{\iota} \dots f_1^{v_2} \frac{u_1^{v_1}}{\iota} f_0^{v_1} \frac{u_0^{v_0}}{\iota} x_0 \\
g_t \frac{u_t^{v_t}}{\iota} x_t &= g_t \frac{u_t^{v_t}}{\iota} f_{t-1}^{v_t} \frac{u_{t-1}^{v_{t-1}}}{\iota} f_{t-2}^{v_{t-1}} \frac{u_{t-2}^{v_{t-2}}}{\iota} \dots f_1^{v_2} \frac{u_1^{v_1}}{\iota} f_0^{v_1} \frac{u_0^{v_0}}{\iota} x_0 \\
J_s^{v_s}(x_s) &= \max_{(u_t)_s} \sum_t^{s|T} \sum_{\omega} \left(g_t \frac{u_t^{V_t^\omega}}{\iota} f_{t-1}^{V_t^\omega} \frac{u_{t-1}^{V_{t-1}^\omega}}{\iota} f_{t-2}^{V_{t-1}^\omega} \frac{u_{t-2}^{V_{t-2}^\omega}}{\iota} \dots f_{s+1}^{V_{s+2}^\omega} \frac{u_{s+1}^{V_{s+1}^\omega}}{\iota} f_s^{V_{s+1}^\omega} \frac{u_s^{V_s^\omega}}{\iota} x_s \right) \\
J_0^{v_0}(x_0) &= \max_{(u_t)_0} \sum_t^{0|T} \sum_{\omega} \left(g_t \frac{u_t^{V_t^\omega}}{\iota} f_{t-1}^{V_t^\omega} \frac{u_{t-1}^{V_{t-1}^\omega}}{\iota} f_{t-2}^{V_{t-1}^\omega} \frac{u_{t-2}^{V_{t-2}^\omega}}{\iota} \dots f_1^{V_2^\omega} \frac{u_1^{V_1^\omega}}{\iota} f_0^{V_1^\omega} \frac{u_0^{v_0}}{\iota} x_0 \right) \\
J_s^{v_s}(x_s) &= \max_{u_s} g_s \frac{u_s^{v_s}}{\iota} x_s + \sum_{\omega}^{\Omega} J_{s+1}^{V_{s+1}^\omega} \left(f_s^{V_{s+1}^\omega} \frac{u_s^{v_s}}{\iota} x_s \right)
\end{aligned}$$