$$\mathbb{K}_{n+1}\triangledown\mathfrak{b} \xleftarrow{\ {}_i\bar{\mathfrak{d}}\ } \mathbb{K}_{n}\triangledown\mathfrak{b}$$

$$\begin{cases} {}_i\bar{\mathfrak{d}}\ {}_J\mathfrak{d} = {}_{iJ}\mathfrak{d} & i \leqslant J \\ {}_i\bar{\mathfrak{d}}\ {}_J\mathfrak{d} - {}_{iJ}\mathfrak{d} \in \mathbb{K}_{n}\triangledown\mathfrak{b} & \\ {}_i\bar{\mathfrak{d}}\ {}_j\bar{\mathfrak{d}}\ {}_K\mathfrak{d} - {}_j\bar{\mathfrak{d}}\ {}_i\bar{\mathfrak{d}}\ {}_K\mathfrak{d} = \overline{{}_i\mathfrak{d} \times {}_j\mathfrak{d}}\ {}_K\mathfrak{d} & |K| < n \end{cases}$$

$$\mathbb{K}_{n+2}\triangledown\mathfrak{b} \xleftarrow{\ {}_i\bar{\bar{\mathfrak{d}}}\ } \mathbb{K}_{n+1}\triangledown\mathfrak{b}$$

$${}_i\bar{\bar{\mathfrak{d}}}\ {}_{jK}\mathfrak{d} = \begin{cases} {}_{ijK}\mathfrak{d} & i \leqslant j \leqslant K \\ {}_{ijK}\mathfrak{d} + \overline{{}_i\mathfrak{d} \times {}_j\mathfrak{d}}\ {}_K\mathfrak{d} + {}_j\bar{\mathfrak{d}}\ \underbrace{{}_i\bar{\mathfrak{d}}\ {}_K\mathfrak{d} - {}_{iK}\mathfrak{d}} & i > j \leqslant K \end{cases}$$

$$L<n \Longrightarrow {}_i\bar{\bar{\eth}}\;{}_j\bar{\eth}\;{}_{kL}\eth - {}_j\bar{\bar{\eth}}\;{}_i\bar{\eth}\;{}_{kL}\eth = \overline{{}_i\eth \times {}_j\eth}\;{}_{kL}\eth$$

$$j>k \Longrightarrow {}_j\bar{\eth}\;{}_{kL}\eth = {}_j\bar{\eth}\;{}_k\bar{\eth}\;{}_L\eth = {}_k\bar{\eth}\;{}_j\bar{\eth}\;{}_L\eth + \overline{{}_j\eth \times {}_k\eth}\;{}_L\eth = {}_k\bar{\eth}\;{}_{jL}\eth + {}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_L\eth$$

$$= {}_{kjL}\eth + {}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_L\eth$$

$$i>k<j \Longrightarrow {}_i\bar{\bar{\eth}}\;{}_{kjL}\eth = {}_{ikjL}\eth + \overline{{}_i\eth \times {}_k\eth}\;{}_{jL}\eth + {}_k\bar{\eth}\;\underbrace{{}_i\bar{\eth}\;{}_{jL}\eth - {}_{ijL}\eth}$$

$${}_i\bar{\bar{\eth}}\;{}_j\bar{\eth}\;{}_{kL}\eth = {}_i\bar{\bar{\eth}}\;{}_{kjL}\eth + {}_i\bar{\eth}\;{}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + {}_i\bar{\eth}\;\overline{{}_j\eth \times {}_k\eth}\;{}_L\eth = {}_{ikjL}\eth + \overline{{}_i\eth \times {}_k\eth}\;{}_{jL}\eth + {}_k\bar{\eth}\;\underbrace{{}_i\bar{\eth}\;{}_{jL}\eth - {}_{ijL}\eth}$$

$$+ {}_k\bar{\eth}\;{}_i\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_i\eth \times {}_k\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_i\bar{\eth}\;{}_L\eth + \overline{{}_i\eth \times \underbrace{{}_j\eth \times {}_k\eth}}\;{}_L\eth$$

$$= {}_{ikjL}\eth + \overline{{}_i\eth \times {}_k\eth}\;{}_j\bar{\eth}\;{}_L\eth + {}_k\bar{\eth}\;\underbrace{{}_i\bar{\eth}\;{}_j\bar{\eth}\;{}_L\eth - {}_{ijL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_i\bar{\eth}\;{}_L\eth + \overline{{}_i\eth \times \underbrace{{}_j\eth \times {}_k\eth}}\;{}_L\eth$$

$$\text{LHS} = {}_k\bar{\eth}\;\underbrace{{}_i\bar{\eth}\;{}_j\bar{\eth} - {}_j\bar{\eth}\;{}_i\bar{\eth}}\;{}_L\eth + \overline{{}_i\eth \times \underbrace{{}_j\eth \times {}_k\eth}}\;{}_L\eth - \overline{{}_j\eth \times \underbrace{{}_i\eth \times {}_k\eth}}\;{}_L\eth$$

$$= {}_k\bar{\eth}\;\overline{{}_i\eth \times {}_j\eth}\;{}_L\eth + \overline{\underbrace{{}_i\eth \times {}_j\eth} \times {}_k\eth}\;{}_L\eth = \overline{{}_i\eth \times {}_j\eth}\;{}_k\bar{\eth}\;{}_L\eth = \text{RHS}$$

$$i \leqslant k<j \Longrightarrow {}_i\bar{\bar{\eth}}\;{}_j\bar{\eth}\;{}_{kL}\eth = {}_i\bar{\bar{\eth}}\;{}_{kjL}\eth + {}_i\bar{\eth}\;{}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + {}_i\bar{\eth}\;\overline{{}_j\eth \times {}_k\eth}\;{}_L\eth$$

$$= {}_{ikjL}\eth + {}_i\bar{\eth}\;{}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + {}_i\bar{\eth}\;\overline{{}_j\eth \times {}_k\eth}\;{}_L\eth$$

$${}_j\bar{\bar{\eth}}\;{}_i\bar{\eth}\;{}_{kL}\eth = {}_j\bar{\bar{\eth}}\;{}_{ikL}\eth = {}_{jikL}\eth + \overline{{}_j\eth \times {}_i\eth}\;{}_{kL}\eth + {}_i\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_{kL}\eth - {}_{jkL}\eth}$$

$$\text{LHS} = \overline{{}_i\eth \times {}_j\eth}\;{}_{kL}\eth + {}_i\bar{\eth}\;\overbrace{{}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_L\eth - {}_j\bar{\eth}\;{}_{kL}\eth + {}_{jkL}\eth}^{=0}$$

$${}_k\bar{\eth}\;\underbrace{{}_j\bar{\eth}\;{}_L\eth - {}_{jL}\eth} + \overline{{}_j\eth \times {}_k\eth}\;{}_L\eth - {}_j\bar{\eth}\;{}_{kL}\eth + {}_{jkL}\eth = {}_j\bar{\eth}\;{}_k\bar{\eth}\;{}_L\eth - {}_j\bar{\eth}\;{}_{kL}\eth - {}_k\bar{\eth}\;{}_{jL}\eth + {}_{jkL}\eth = 0$$