$$\prod_n^{\prec} \underbrace{\mathbb{Z} \ulcorner \mathbb{Z}n} = \frac{(x_n + \mathbb{Z}n)_n}{m \prec n \curvearrowright m \prec x_m - x_n} \subset \prod_n \underbrace{\mathbb{Z} \ulcorner \mathbb{Z}n}_{\text{fin}} \text{ cpt}$$

$$\overset{p}{\mathbb{Z}} = \prod_i^{\prec} \underbrace{\mathbb{Z} \ulcorner \mathbb{Z}p^i} = \frac{\left(x_i + \mathbb{Z}p^i\right)_i}{i \leqslant j \curvearrowright p^i \prec x_i - x_j} \subset \prod_i \underbrace{\mathbb{Z} \ulcorner \mathbb{Z}p^i}$$

$$\overset{p}{\mathbb{Z}} = \frac{\sum_i^{\mathbb{N}} a_i p^i}{a_i \in p}$$

$$0 \leqslant x_n < p^n$$

$$x_n = \sum_i^n a_i \, p^i$$

$$\underline{n = 1} \ 0 \leqslant x_1 < p^1 = p \Longrightarrow a_0 = x_1 \in p \Longrightarrow x_1 = a_0 \, p^0$$

$$\underline{1 \leqslant n \curvearrowright n + 1} \ x_n = \sum_i^n a_i \, p^i$$

$$0 \leqslant x_{n+1} < p^{n+1}$$

$$p^n \prec x_{n+1} - x_n \Longrightarrow x_{n+1} - x_n = a_n \, p^n$$

$$\begin{cases} x_{n+1} \geqslant p^n > x_n & \Longrightarrow 0 < x_{n+1} - x_n = a_n p^n < p^{n+1} \Longrightarrow 0 < a_n < p \\ x_{n+1} < p^n > x_n & \Longrightarrow \ulcorner a_n \urcorner p^n = \ulcorner x_{n+1} - x_n \urcorner < p_n \Longrightarrow a_n = 0 \end{cases}$$

$$\prod_n^{\prec} \underbrace{\mathbb{Z} \ulcorner \mathbb{Z}n} = \prod_p \overset{p}{\mathbb{Z}}$$