

$$\mathbb{U}:\ast \text{ halbgrp}$$

$$\text{komm } \mathbb{U} \supset Q \text{ kuerzbar} \Leftrightarrow \bigwedge_{\dot{m}} \bigwedge_{\dot{p}}^{\mathbb{U} \quad Q} m \ast p = n \ast p \Rightarrow m = n$$

$$m:p \sim n:q \Leftrightarrow m \ast q = n \ast p \text{ equ rel } \mathbb{U} \sqcup Q$$

$$\text{refl } m \ast p = m \ast p \Rightarrow m:p \sim m:p$$

$$\text{symm } m:p \sim n:q \Rightarrow m \ast q = n \ast p \Rightarrow n \ast p = m \ast q \Rightarrow n:q \sim m:p$$

$$\text{trans } m:p \sim n:q \sim r:s \Rightarrow \begin{cases} m \ast q = n \ast p \\ n \ast s = r \ast q \end{cases} \Rightarrow \underline{m \ast s} \ast q = m \ast \underline{n \ast q}$$

$$\begin{aligned} & \stackrel{=}{\text{komm}} m \ast \underline{q \ast s} = \underline{m \ast q} \ast s = \underline{n \ast p} \ast s \stackrel{=}{\text{komm}} \underline{p \ast n} \ast s = p \ast \underline{n \ast s} = p \ast \underline{r \ast q} = \underline{p \ast r} \ast q \stackrel{=}{\text{komm}} \underline{r \ast p} \ast q \\ & \stackrel{\Rightarrow}{\text{cancel}} m \ast s = r \ast p \Rightarrow m:p \sim r:s \end{aligned}$$

$$m \wr p = \frac{n:q \in \mathbb{U} \sqcup Q}{n:q = m:p} = \frac{n:q \in \mathbb{U} \sqcup Q}{n \ast p = m \ast q}$$

$$\mathbb{U} \wr Q = \frac{m \wr p}{m \in \mathbb{U}: \quad p \in Q}$$

$$\mathbb{U} \wr Q \text{ abel semi-grp}$$

$$\underline{m \wr p} \underline{n \wr q} = \underline{m \ast n} \wr \underline{p \ast q}$$

$$\begin{aligned} \dot{m}:\dot{p} \sim \dot{n}:\dot{q} &\Rightarrow \dot{m} \ast \dot{q} = \dot{n} \ast \dot{p} \Rightarrow \underline{m \ast n} \ast \underline{q \ast \dot{q}} = \underline{m \ast n} \ast \underline{\dot{q} \ast q} = m \ast \underline{n \ast \dot{q}} \ast q \\ &= m \ast \underline{\dot{n} \ast q} \ast q = \underline{m \ast \dot{n}} \ast \underline{q \ast q} = \underline{\dot{n} \ast m} \ast \underline{q \ast q} = \dot{n} \ast \underline{m \ast q} \ast q = \dot{n} \ast \underline{n \ast p} \ast q \\ &= \underline{\dot{n} \ast n} \ast \underline{p \ast q} = \underline{n \ast \dot{n}} \ast \underline{p \ast q} \Rightarrow \underline{m \ast n}:\underline{p \ast q} \sim \underline{n \ast \dot{n}}:\underline{q \ast \dot{q}} \Rightarrow \ast \text{ well-def} \end{aligned}$$

$$\text{neutral } o \in Q \subset \mathbb{U} \Rightarrow \text{neutr } o \wr o \in \mathbb{U} \wr Q$$

$$\underline{m \wr p} \ast \underline{o \wr o} = \underline{m \ast o} \wr \underline{p \ast o} = m \wr p = \underline{o \ast m} \wr \underline{o \ast p} = \underline{o \wr o} \ast \underline{m \ast p}$$

$$\mathbb{U}:\ast :o \text{ unit abel halbgrp} \Rightarrow \mathbb{U} \wr \mathbb{U} \text{ abel grp}$$

$$\overbrace{m \wr p}^{-1} = p \wr m$$

$$\underbrace{m \wr p} \ast \underbrace{p \wr m} = \underbrace{m \ast p} \wr \underbrace{p \ast m} = o \wr o \Leftarrow \underbrace{m \ast p} \ast o = o \ast \underbrace{p \ast m}$$

$$\mathbb{U} \rightarrow \mathbb{U} \wr Q$$

$$m \rightarrow m \wr o$$

$$\underbrace{m \ast n} \wr o = \underbrace{m \wr o} \ast \underbrace{n \wr o}$$

$$\underbrace{m \wr o} \ast \underbrace{n \wr o} = \underbrace{m \ast n} \wr \underbrace{o \ast o} = \underbrace{m \ast n} \wr o$$