$$\mathbb{I} \supset \mathbb{U} \text{ semigroup}$$

$$\mathbb{U} \ni u \text{ unit}$$

$$\text{fraction relation } R \text{ on } \mathbb{I}\times\mathbb{U} = \frac{m{:}p}{m \in \mathbb{I}{:}\quad p \in \mathbb{U}}$$

$$m{:}p \sim n{:}q \Leftrightarrow \bigvee_{b \in \mathbb{U}} \underbrace{m{*}q}{*}b = \underbrace{n{*}p}{*}b \text{ equ rel}$$

$$\text{refl } m{*}p = m{*}p \Longrightarrow m{:}p \sim m{:}p$$

$$\text{symm } m{:}p \sim n{:}q \Longrightarrow m{*}q{*}b = n{*}p{*}b \Longrightarrow n{*}p{*}b = m{*}q{*}b \Longrightarrow n{:}q \sim m{:}p$$

$$\text{trans } m{:}p \sim n{:}q \sim r{:}s \Longrightarrow \begin{cases} \bigvee\limits_{b \in \mathbb{U}} & m{*}q{*}b = n{*}p{*}b \\ \bigvee\limits_{d \in \mathbb{U}} & n{*}s{*}d = r{*}q{*}d \end{cases}$$

$$\Longrightarrow \underbrace{m{*}s}{*}\underbrace{q{*}b{*}d}{} = \underbrace{m{*}q{*}b}{*}\underbrace{s{*}d}{} \underset{\text{Vor}}{=} \underbrace{n{*}p{*}b}{*}\underbrace{s{*}d}{}$$

$$= \underbrace{n{*}s{*}d}{*}\underbrace{p{*}b}{} \underset{\text{Vor}}{=} \underbrace{r{*}q{*}d}{*}\underbrace{p{*}b}{} = \underbrace{r{*}p}{*}\underbrace{q{*}b{*}d}{} \underset{q{*}b{*}d \in \mathbb{U}}{\Longrightarrow} m{:}p \sim r{:}s$$

$$\text{fraction class } m \circledast p$$

$$m{:}p \sim n{:}q \Leftrightarrow m \circledast p = n \circledast q$$

$$\mathbb{I} \circledast \mathbb{U} = \underbrace{\mathbb{I}\times\mathbb{U}}{} \sqsubset R = \frac{m \circledast p}{m \in \mathbb{I}{:}\quad p \in \mathbb{U}}$$

$$m \circledast p \in \mathbb{I} \circledast \mathbb{U} \xleftarrow[\text{surj}]{\circledast} \mathbb{I}\times\mathbb{U} \ni m{:}p$$

composition of fractions $\underbrace{\mathbb{1} \circledast \mathbb{4}} \times \underbrace{\mathbb{1} \circledast \mathbb{4}} \xrightarrow{*} \mathbb{1} \circledast \mathbb{4}$

$\underbrace{m \circledast p} * \underbrace{n \circledast q} = \underbrace{m * n} \circledast \underbrace{p * q}$ easy well-def

$$\begin{cases} m \circledast p = \acute{m} \circledast \acute{p} & \Longrightarrow m * \acute{p} * b = \acute{m} * p * b \\ n \circledast q = \acute{n} \circledast \acute{q} & \Longrightarrow n * \acute{q} * d = \acute{n} * q * d \end{cases} \Longrightarrow$$

$$\overbrace{\underbrace{m * n} * \underbrace{\acute{p} * \acute{q}}} * \underbrace{b * d} = \underbrace{m * \acute{p} * b} * \underbrace{n * \acute{q} * d} \underset{\text{Vor}}{=} \underbrace{\acute{m} * p * b} * \underbrace{\acute{n} * q * d} = \overbrace{\underbrace{\acute{m} * \acute{n}} * \underbrace{p * q}} * \underbrace{b * d}$$

$$\underset{b * d \in \mathbb{4}}{\overset{\text{def}}{\Longrightarrow}} \underbrace{m * n} : \underbrace{p * q} \sim \underbrace{\acute{m} * \acute{n}} : \underbrace{\acute{p} * \acute{q}} \Longrightarrow \underbrace{m * n} \circledast \underbrace{p * q} = \underbrace{\acute{m} * \acute{n}} \circledast \underbrace{\acute{p} * \acute{q}}$$

trivial fraction $m \circledast u \in \mathbb{1} \circledast \mathbb{4}$ $\xleftarrow[\circledast]{} \mathbb{1} \times \mathbb{4} \xleftarrow[:u]{} \mathbb{1} \ni m$

$\circledast u$

$\mathbb{4}$ kuerzbar $\Longrightarrow$ inj

$\underbrace{m \circledast u} * \underbrace{m \circledast u} = \underbrace{m * n} \circledast u$