$$
\begin{tikzcd}
\mathbb{1}^{\sharp} \llcorner \mathbb{1}^{1} = \mathbb{1}^{\sharp} \llcorner \mathbb{1} \arrow[d, "d"'] \arrow[dd, bend right=60, "d^2 = 0"'] \\
\mathbb{1}^{\sharp} \llcorner \mathbb{1}^{2} \arrow[d, "d"'] \\
\mathbb{1}^{\sharp} \llcorner \mathbb{1}^{3}
\end{tikzcd}
$$

$$dd = 0$$

$$d\,d\,\overbrace{\underbrace{\mathbb{1}^{j} \boxtimes_{j} \mathbb{1}}} = d\,\overbrace{\underbrace{1 \boxtimes \mathbb{1}^{j} - \mathbb{1}^{j} \boxtimes 1} \mathbin{\blacklozenge} \overbrace{1 \boxtimes_{j} \mathbb{1} - {}_{j}\mathbb{1} \boxtimes 1}}$$

$$= \overbrace{d\,\underbrace{1 \boxtimes \mathbb{1}^{j} - \mathbb{1}^{j} \boxtimes 1}}^{=0} \mathbin{\blacklozenge} \overbrace{1 \boxtimes_{j} \mathbb{1} - {}_{j}\mathbb{1} \boxtimes 1} + \underbrace{1 \boxtimes \mathbb{1}^{j} - \mathbb{1}^{j} \boxtimes 1} \mathbin{\blacklozenge} \overbrace{d\,\underbrace{1 \boxtimes_{j} \mathbb{1} - {}_{j}\mathbb{1} \boxtimes 1}}^{=0} = 0$$