

$${}_{\scriptscriptstyle e}\mathfrak{S}^{\scriptscriptstyle O}|_{{\scriptscriptstyle v}\mathbb{L}}=\mathfrak{S}^{\scriptscriptstyle O}_1|_{{\scriptscriptstyle v}\mathbb{L}}\times \mathfrak{S}^{\scriptscriptstyle O}_{-}|_{{\scriptscriptstyle v}\mathbb{L}}\times \sum_{1\leqslant i < j\leqslant r}\mathfrak{S}^{\scriptscriptstyle \kappa}_j-i|_{{\scriptscriptstyle v}\mathbb{L}}\times \sum_{0\leqslant i\leqslant j\leqslant r}\mathfrak{S}^{\scriptscriptstyle 1}_j+i|_{{\scriptscriptstyle v}\mathbb{L}}$$

$$e\cdot X_a^\varkappa=a-e\mathring a e+\varkappa|\{i:j\}|\left(e\mathring a e_j-e\mathring e_j a\right)=\begin{cases}a-\mathring a+2\varkappa(\mathring a/2-a/2)=(1-\varkappa)(1-\varepsilon)a&1\leqslant i< j\leqslant r\\a-\mathring a+\varkappa(\mathring a-a)=(1-\varkappa)(1-\varepsilon)a&1\leqslant i=j\leqslant r\\a+2\varkappa(-a/2)=(1-\varkappa)a&0=i< j\leqslant r\end{cases}$$

$$e\delta = \sum_k e_k \delta = 0$$

$$e_k - e\mathring e_k e = e_k - e_k = 0$$

$$\textcolor{blue}{\cancel{\star} A}_{\mathcal{G}_e}=-\frac{2n}{r}\tau\left(e\cdot d_eA\right)$$

$$A=\sum_k\lambda^kX_{e_k}^-\Rightarrow \textcolor{blue}{\cancel{\star} A}_{\mathcal{G}_e}=\underbrace{\cancel{\star}\sum_k\lambda^k\widehat{e_k-\mathring e_k}}_{\mathcal{G}_e}=\frac{2n}{r}\sum_j\lambda^j$$

$$\begin{aligned}\text{LHS } &= \sum_k\lambda^k\underbrace{\cancel{\star} e_k-\mathring e_k}_{\mathcal{G}_e}=\pm\sum_{1\leqslant i < j\leqslant r}a\underbrace{\lambda^j-\lambda^i}_{\phantom{\lambda^j-\lambda^i}}+\sum_{1\leqslant i < j\leqslant r}a\underbrace{\lambda^j+\lambda^i}_{\phantom{\lambda^j+\lambda^i}}+\sum_{1\leqslant j\leqslant r}\underline{2\lambda^j+2b\lambda^j}\\&= \sum_{1\leqslant i < j\leqslant r}a\underbrace{\lambda^j+\lambda^i}_{\phantom{\lambda^j+\lambda^i}}+\sum_{1\leqslant j\leqslant r}2(1+b)\lambda^j=a\begin{bmatrix}\lambda^2+\lambda^1+\lambda^3+\lambda^1+\cdots+\lambda^r+\lambda^1\\\lambda^3+\lambda^2+\cdots+\lambda^r+\lambda^2\\\lambda^r+\lambda^{r-1}\end{bmatrix}+2(1+b)\sum_{1\leqslant j\leqslant r}\lambda^j=\\&\quad =\sum_j\lambda^j\left(a(r-1)+2(1+b)\right)=\text{ RHS}\end{aligned}$$

$$d_eA=-\sum_k\lambda^kze_k^*e\Rightarrow e\cdot d_eA=-\sum_k\lambda^ke_k\underbrace{\cancel{\star}\sum_k\lambda^k\widehat{e_k-\mathring e_k}}_{\mathcal{G}_e}=\frac{2n}{r}\sum_j\lambda^j$$

$$\Delta_G\left(g\right)=\Delta\left(e\cdot d_eg\right)^{2n/r}$$