

$$P \leftarrow \mathbb{H} \ltimes P$$

$${^p\widehat{\textcolor{red}{\mathcal{L}} \ltimes \mathcal{G}}} = \underbrace{\textcolor{blue}{\mathcal{L}} \ltimes}_{p} {^p\mathcal{G}} = {}^0\partial_t^{{}^t\mathfrak{e}^t \ltimes p}\mathcal{G}$$

$$P \xrightarrow{\pi} \mathbb{H} = \mathbb{H} \curvearrowright P$$

$$\dot{p} \in \underline{P}_p \xrightarrow{\bar{\pi}} \mathbb{H}_x \ni \dot{x}$$

$$\dot{p} \in \underline{P}_p \xrightarrow{p_{\mathfrak{q}}} \mathbb{H} \ni \dot{p}^{p_{\mathfrak{q}}}$$

$$\dot{p} - \underbrace{\dot{p}^{p_{\mathfrak{q}} \ltimes}}_p \text{ horiz lift of } \dot{p}^{p_{\underline{\pi}}} = \dot{x}$$

$$\begin{aligned} \dot{p} - \underbrace{\dot{p}^{p_{\mathfrak{q}} \ltimes}}_p &\in \underline{P}_p^= \text{ horiz} \\ \overbrace{\dot{p} - \underbrace{\dot{p}^{p_{\mathfrak{q}} \ltimes}}_p}^{p_{\underline{\pi}}} &= \dot{p}^{p_{\underline{\pi}}} - \overbrace{\underbrace{\dot{p}^{p_{\mathfrak{q}} \ltimes}}_p}^{=0} = \dot{p}^{p_{\underline{\pi}}} \end{aligned}$$