$$P \leftarrow \mathbb{h} \ltimes P$$

$$^{p}\overbrace{\flat \ltimes \P} = \underbrace{\flat \ltimes}_{p} {}^{p}\underline{\P} = {}^{0}\partial_{t}{}^{t\mathfrak{e}^{\flat} \ltimes p}\P$$

$$P \xrightarrow{\pi} \hbar = \mathbb{h} \urcorner P$$

$$\dot{p} \in \underline{P}_{p} \xrightarrow{{}^{p}\underline{\pi}} \underline{\hbar}_{x} \ni \dot{x}$$

$$\dot{p} \in \underline{P}_{p} \xrightarrow{{}^{p}\P} \underline{\mathbb{h}} \ni \dot{p}\,{}^{p}\P$$

$$\dot{p} - \underbrace{\dot{p}^{p}\P\ltimes}_{p} \text{ horiz lift of } \dot{p}\,{}^{p}\underline{\pi} = \dot{x}$$

$$\dot{p} - \underbrace{\dot{p}^{p}\P\ltimes}_{p} \in \underline{P}_{p}^{=} \text{ horiz}$$

$$\overbrace{\dot{p} - \underbrace{\dot{p}^{p}\P\ltimes}_{p}}\,{}^{p}\underline{\pi} = \dot{p}\,{}^{p}\underline{\pi} - \overbrace{\underbrace{\dot{p}^{p}\P\ltimes}_{p}\,{}^{p}\underline{\pi}}^{=0} = \dot{p}\,{}^{p}\underline{\pi}$$