

$$(C)_n=\sum_{\mu\in r\times n}(K)_\mu$$

$$K\ltimes {}^Z\mathbb{D}_{\bullet}^n=\sum_{|\mu|\leq n} {}^Z\mathbb{D}_{\bullet}^{\mu}$$

$${}^Z\mathbb{D}_{\bullet}^n\cong \zeta K_{-\omega}^n=\sum_{n\geq |\mu}(-n)_{\mu}\zeta K_{-\omega}^{\mu}=\sum_{n\geq |\mu}(-n)_{\mu}1^{\mu}\underbrace{\zeta K_{\omega}^{\mu}}_{\in {}^Z\mathbb{D}_{\bullet}^{\mu}}$$