

$$S_{1\cdots r}=\frac{u_1\cdots u_r}{u_j\in S_j:u_1<\cdots <u_r}$$

$$S_{1\cdots r}\overset{k}{\rightarrow}S_{1\cdots r}$$

$$K_{1\cdots r}\lhd K\overset{\sim}{\rightarrow}S_{1\cdots r}$$