

$$\begin{aligned}
\Delta \frac{a}{c} \Big| \frac{b}{d} &= \frac{a}{c} \Big| \frac{b}{d} \mathbf{\Sigma} \frac{a}{c} \Big| \frac{b}{d} = \frac{a\mathbf{\Sigma}a + b\mathbf{\Sigma}c}{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} \Big| \frac{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d}{c\mathbf{\Sigma}b + d\mathbf{\Sigma}d} \\
\Delta \underline{ab - qba} &= \underline{\Delta a \mathbf{\Sigma} \Delta b} - q \underline{\Delta b \Delta a} = \underline{a\mathbf{\Sigma}a + b\mathbf{\Sigma}c} \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} - q \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} \underline{a\mathbf{\Sigma}a + b\mathbf{\Sigma}c} \\
&= aa \mathbf{\Sigma}_* ab + ab \mathbf{\Sigma} ad + ba \mathbf{\Sigma} cb + bb \mathbf{\Sigma}_\# cd - qaa \mathbf{\Sigma}_* ba - qab \mathbf{\Sigma} bc - qba \mathbf{\Sigma} da - qbb \mathbf{\Sigma}_\# dc \\
&= aa \mathbf{\Sigma}_* \underbrace{ab - qba}_{\in \mathcal{I}} + bb \mathbf{\Sigma}_\# \underbrace{cd - qdc}_{\in \mathcal{I}} + \underbrace{ab - qba}_{\in \mathcal{I}} \mathbf{\Sigma} \underbrace{da - q^{-1}cb}_{\in \mathcal{I}} + ab \mathbf{\Sigma} \underbrace{ad - qbc - da + q^{-1}cb}_{\in \mathcal{I}} \\
\Delta \underline{bc - cb} &= \underline{\Delta b \mathbf{\Sigma} \Delta c} - \underline{\Delta c \Delta b} = \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} - \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} \\
&= ac \mathbf{\Sigma}_* ba + ad \mathbf{\Sigma}_+ bc + bc \mathbf{\Sigma}_- da + bd \mathbf{\Sigma}_\# dc - ca \mathbf{\Sigma}_* ab - cb \mathbf{\Sigma}_- ad - da \mathbf{\Sigma}_+ cb - db \mathbf{\Sigma}_\# cd \\
&= \underline{ac - qca} \mathbf{\Sigma}_* ba - ca \mathbf{\Sigma}_* \underline{ab - qba} + \underline{bd - qdb} \mathbf{\Sigma}_\# dc - db \mathbf{\Sigma}_\# \underline{cd - qdc} \\
&+ \underbrace{ad - qbc - da + q^{-1}cb}_{\in \mathcal{I}} \mathbf{\Sigma}_+ bc + da \mathbf{\Sigma}_+ \underline{bc - cb} - cb \mathbf{\Sigma}_- \underline{ad - qbc - da + q^{-1}cb} + \underline{bc - cb} \mathbf{\Sigma}_- da \\
&\quad + \underline{qbc - cb} \mathbf{\Sigma} bc + q^{-1}cb \mathbf{\Sigma} \underline{cb - bc} \\
\Delta \underline{ad - qbc - da + q^{-1}cb} &= \Delta a \Delta d - q \Delta b \Delta c - \Delta d \Delta a + q^{-1} \Delta c \Delta b \\
&= \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} - q \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} - \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d} + q^{-1} \underline{c\mathbf{\Sigma}a + d\mathbf{\Sigma}c} \underline{a\mathbf{\Sigma}b + b\mathbf{\Sigma}d}
\end{aligned}$$

$$ba = qab: \quad ca = qac: \quad db = qbd: \quad dc = qcd: \quad bc = cb: \quad ad - q^{-1}bc = da - qbc$$

$$KE = q^2 EK$$

$$KF = q^{-2} FK$$

$$EF - FE = \frac{K - K^{-1}}{q - q^{-1}}$$

$$\Delta E = 1 \mathbf{\Sigma} E + E \mathbf{\Sigma} K$$

$$\Delta F = K^{-1} \mathbf{\Sigma} F + F \mathbf{\Sigma} 1$$

$$E|b = 1 = F|c: \quad K|a = q: \quad K|d = q^{-1}$$

$$\deg E^i F^j K^\ell = i \vee j$$

$$\deg P \geq 3 \Rightarrow \Delta P = P_\alpha \mathbf{\Sigma}^\alpha P \Rightarrow \deg P_\alpha \geq 2 \Rightarrow P_\alpha \Big| \frac{a}{c} \Big| \frac{b}{d} = 0 \Rightarrow \Delta P | {}^i g_j {}^k g_\ell = 0$$

$$\Rightarrow \text{comm relations for } {}^i g_j {}^k g_\ell$$

$$\deg P \leq 2$$

K^ℓ	1	F	F^2
1	$ad = 1 = da$	$\begin{array}{c} c \ a = q^{2\ell} = q \ a \ c \\ d \ c = q = q \ c \ d \end{array}$	0
E	$\begin{array}{c} b \ a = q = q \ a \ b \\ d \ b = q^{-2\ell} = q \ b \ d \end{array}$	$\begin{array}{c} bc = 1 = cb \\ da = q = qad \end{array}$	$ca = \alpha_2 q^{2\ell-1} = qac$
E^2	0	$ba = \alpha_4 = qab$	0