

$${}_z\varphi_{\zeta}=\frac{\zeta-z}{1-z\bar{\zeta}}$$

$$b\in\mathbb{C}\Rightarrow {}_z\gamma_b=1|z\frac{1}{b^*}\bigg|1=\frac{b+z}{1+z\bar{b}^*}\Rightarrow {}_0\gamma_{-b}=1-\overline{b}^2$$

$$\frac{1}{b^*}\bigg|\frac{-1}{b}\bigg|1=\frac{1}{-b^*}\bigg|\frac{-b}{1}\bigg|\Rightarrow \gamma_b^{-1}=\gamma_{-b}$$

$$k^{-1}\,\gamma_b\,k=\gamma_{bk}$$

$${}_z\sigma_b=-{}_z\gamma_{-b}=-\gamma_b^{-1}$$

$${}_0\sigma_b=-{}_0\gamma_{-b}= -(-b)=b$$

$${}_b\sigma_b=-{}_b\gamma_{-b}=-{}_b\gamma_b^{-1}=-0=0$$

$$\sigma_b\,\sigma_b=\imath$$

$$\sigma_b\,\sigma_b=\gamma_{-b}\,(-\imath)\,\gamma_{-b}\,(-\imath)=\gamma_{-b}\,\gamma_b=\imath$$