

$$\Delta = \underbrace{1 + z\bar{z}}^2 \partial_z \bar{\partial}_z$$

$$\sigma^2 = 1$$

$${}_z\mathbf{J} = \frac{1 + z\bar{z}}{\underbrace{z + \sigma} \underbrace{\bar{z} - \sigma}}$$

$$\Delta \mathbf{J}^\alpha = \alpha \underbrace{1 - \alpha} \mathbf{J}^\alpha$$

$$\partial_z \mathbf{J} = \frac{\sigma}{\underbrace{z + \sigma}^2}$$

$$\bar{\partial}_z \mathbf{J} = \frac{-\sigma}{\underbrace{\bar{z} - \sigma}^2}$$

$$\begin{aligned} \Delta \mathbf{J}^\alpha &= \underbrace{1 + z\bar{z}}^2 \partial_z \bar{\partial}_z \mathbf{J}^\alpha = \underbrace{1 + z\bar{z}}^2 \partial_z \alpha \mathbf{J}^{\alpha-1} \frac{-\sigma}{\underbrace{\bar{z} - \sigma}^2} = -\alpha \sigma \overbrace{\frac{1 + z\bar{z}}{\bar{z} - \sigma}}^2 \partial_z \mathbf{J}^{\alpha-1} \\ &= -\alpha \sigma \overbrace{\frac{1 + z\bar{z}}{\bar{z} - \sigma}}^2 \underbrace{\alpha - 1}_{\alpha - 1} \mathbf{J}^{\alpha-2} \frac{\sigma}{\underbrace{z + \sigma}^2} = \alpha \underbrace{1 - \alpha}_{1 - \alpha} \overbrace{\frac{1 + z\bar{z}}{z + \sigma} \frac{\bar{z} - \sigma}{\bar{z} - \sigma}}^2 \mathbf{J}^{\alpha-2} = \alpha \underbrace{1 - \alpha}_{1 - \alpha} \mathbf{J}^\alpha \end{aligned}$$