$$\mathbb{C} = \mathbb{R} + i\mathbb{R}$$

$$\mathfrak{e}\ \Big\downarrow\ \text{diff}$$

$$\mathbb{C}^{\times} = \mathbb{R}_{>}\mathbb{T}$$

$${}^{x+iy}\mathfrak{e} = {}^{x}\mathfrak{e}\,{}^{iy}\mathfrak{e} = {}^{x}\mathfrak{e}\,\underbrace{{}^{y}\mathfrak{c} + i\,{}^{y}\mathfrak{s}}$$

$${}^{iy}\mathfrak{e} = {}^{y}\mathfrak{c} + i\,{}^{y}\mathfrak{s}$$

$$\mathbb{C} \xrightarrow[\text{hom}]{\mathfrak{e}} \mathbb{C}^{\times}$$

$$i\mathbb{R} \xrightarrow[\text{hom}]{\mathfrak{e}} \mathbb{T}$$

$$\mathbb{T}$$

$$\text{hom}\ \Big\uparrow\ \mathfrak{e}$$

$$i\mathbb{R}$$

$${}^{z+w}\mathfrak{e} = {}^{z}\mathfrak{e}\,{}^{w}\mathfrak{e}$$

$${}^{x+iy}\mathfrak{e}\,{}^{u+iv}\mathfrak{e} = {}^{x}\mathfrak{e}\,\underbrace{{}^{y}\mathfrak{c} + i\,{}^{y}\mathfrak{s}}\,{}^{u}\mathfrak{e}\,\underbrace{{}^{v}\mathfrak{c} + i\,{}^{v}\mathfrak{s}} = {}^{x}\mathfrak{e}\,{}^{u}\mathfrak{e}\,\underbrace{{}^{y}\mathfrak{c} + i\,{}^{y}\mathfrak{s}}\,\underbrace{{}^{v}\mathfrak{c} + i\,{}^{v}\mathfrak{s}}$$

$$\underset{\text{Add}}{=}\ {}^{x}\mathfrak{e}\,{}^{u}\mathfrak{e}\,\overbrace{\underbrace{{}^{y}\mathfrak{c}\,{}^{v}\mathfrak{c} - {}^{y}\mathfrak{s}\,{}^{v}\mathfrak{s}}_{= {}^{y+v}\mathfrak{c}} + i\underbrace{{}^{y}\mathfrak{s}\,{}^{v}\mathfrak{c} + {}^{y}\mathfrak{c}\,{}^{v}\mathfrak{s}}_{= {}^{y+v}\mathfrak{s}}} = {}^{x+u}\mathfrak{e}\,\underbrace{{}^{y+v}\mathfrak{c} + i\,{}^{y+v}\mathfrak{s}} = {}^{z+w}\mathfrak{e}$$

$$\text{Ker } \mathfrak{e} = 2i\pi\mathbb{Z} = \frac{2\pi i n}{\mathbb{Z} \ni n}$$

$$t = r + is \in \text{ Ker } \mathfrak{e} \Longrightarrow 1 = {}^{t}\mathfrak{e} \underset{=}{\Longrightarrow} \overline{{}^{t}\mathfrak{e}} = {}^{\Re t}\mathfrak{e} = {}^{r}e \Longrightarrow r = 0$$

$$1 = {}^{is}\mathfrak{e} = {}^{s}\mathfrak{c} + i\,{}^{s}\mathfrak{s} \Longrightarrow \begin{cases} 1 = {}^{s}\mathfrak{c} \\ 0 = {}^{s}\mathfrak{s} \end{cases} \Longrightarrow \bigvee_{n}^{\mathbb{Z}} s = 2\pi i n$$

$$\begin{array}{ccc} i\mathsf{L} & \xrightarrow{\in} & \mathbb{R}_{\infty} \text{ abel} \\ \uparrow{\scriptstyle \mathfrak{e} = \imath} & & \\ \overline{i\mathsf{L}} & \xrightarrow{=} & i\mathsf{L} \end{array}$$

$$i\mathsf{L} \underset{\asymp}{\longrightarrow} \mathbb{T}_{0}\,\mathsf{L}^{\sharp} = \mathbb{T}_{0}\,\mathsf{L}\,\mathbb{R}$$

$$\begin{array}{ccc} i\mathbb{R} & \xrightarrow{\in} & \mathbb{R}_{\infty} \text{ abel} \\ \uparrow{\scriptstyle \exp = id} & & \\ \overline{i\mathbb{R}} & \xrightarrow{=} & i\mathbb{R} \end{array}$$

$$\mathsf{V}_{i\mathsf{L}^{\cdot}} = \frac{d\mathsf{L}^{1} \cdots d\mathsf{L}}{(2\pi)} = \frac{d\mathsf{L}^{1}}{2\pi} \cdots \frac{d\mathsf{L}}{2\pi}$$

$$i\mathbb{R} \xrightarrow{\asymp} \mathbb{T}_{0}\,\mathbb{R} = \mathbb{T}_{0}\,\overbrace{\mathbb{R}\,\mathbb{R}}$$

$$\mathsf{L}^{\cdot} \xrightarrow{\mapsto} \exp \lfloor \dot{.} \rceil \leftarrow . \rceil$$

$$\lambda_{\mathsf{L}^{\cdot}} = d\mathsf{L}^{1} \cdots d\mathsf{L} \in \mathbb{R}_{+} \triangledown_{-0} \mathbb{R}$$

$$\mathbb{R} \xrightarrow{\asymp} \mathbb{T} \triangledown_{0} \overset{>}{\mathbb{R}}$$

$$\mathsf{L}^{\cdot} \xrightarrow{\mapsto} \exp 2\pi i \mathsf{L}^{\cdot} \underbrace{\log . \mathsf{Y}} \leftarrow\!\shortmid . \mathsf{Y}$$

$$i\mathbb{Q}_p \xrightarrow{\asymp} \mathbb{T} \triangledown_{0} \mathbb{Q}_p$$

$$\mathsf{L}^{\cdot} \xrightarrow{\mapsto} \exp 2\pi i \operatorname{tr}_p \mathsf{L}^{\cdot} . \rceil \leftarrow\!\shortmid . \rceil$$

$$is \mapsto e^{is} = {}^{s}\mathfrak{c} + i\, {}^{s}\mathfrak{s}$$

$$\text{Ker } (\exp) = 2\pi i \mathbb{Z} = \frac{2\pi i n}{\mathbb{Z} \ni n}$$

$$is \in \text{ Ker } \mathfrak{e} \Rightarrow 1 = {}^{is}\mathfrak{e} = {}^{s}\mathfrak{c} + i\, {}^{s}\mathfrak{s} \Rightarrow 1 = {}^{s}\mathfrak{c}$$

$$0 = {}^{s}\mathfrak{s} \Rightarrow s = 2\pi i n : \quad n \in \mathbb{Z}$$