$$ {}^{X}\triangle_{\bullet}\mathbb{k} $$

$$ 0 \neq p \in {}^{X}\triangle_{\bullet}\mathbb{k} \underset{\text{DIV}}{\Longrightarrow} \bigwedge_{\daleth}^{{}^{X}\triangle_{\bullet}\mathbb{k}} \bigvee_{q:r} \begin{cases} \daleth = pq + r \\ \underleftarrow{r} < \underleftarrow{p} = m \end{cases} $$

$$ \text{Eind } \daleth = \dot{p}\,\dot{q} + \dot{r} \Longrightarrow r - \acute{r} = p\,(\acute{q} - q) $$

$$ \lightning\; q \neq \acute{q} \Longrightarrow \underleftarrow{p} > \underbrace{r - \acute{r}}_{\leftharpoondown} = \underbrace{p\,(\acute{q} - q)}_{\leftharpoondown} = \underleftarrow{p} + \underbrace{\acute{q} - q}_{\leftharpoondown} \geqslant \underleftarrow{p} \;\lightning $$

$$ \Longrightarrow q = \acute{q} \Longrightarrow r = \acute{r} $$

$$ \text{Ex} \quad \text{Ind } n = \deg \daleth \in \mathbb{N} \cup -\infty $$

$$ \begin{cases} n < m & \Longrightarrow q = 0 \wedge r = \daleth \\ n \geqslant m & \Longrightarrow h := \daleth - \dfrac{{}^{n}\daleth}{{}_{m}p} p X^{n\text{-}m} = \underbrace{X^{n}{}_{n}\daleth + X^{n\text{-}1}{}_{n\text{-}1}\daleth + \cdots + {}_{0}\daleth}_{} - \dfrac{{}^{n}\daleth}{{}_{m}p} \underbrace{X^{n}{}_{m}p + X^{n\text{-}1}{}_{m\text{-}1}p + \cdots + X^{n\text{-}m}{}_{0}p}_{} \\ & \Longrightarrow \deg h < n \underset{\text{Ind}}{\Longrightarrow} h = pg + r \end{cases} $$

$$ \deg r < m \Longrightarrow \daleth = h + \frac{{}^{n}\daleth}{{}_{m}p} X^{n\text{-}m} p = pg + r + \frac{{}^{n}\daleth}{{}_{m}p} X^{n\text{-}m} p = p \underbrace{g + \frac{{}^{n}\daleth}{{}_{m}p} X^{n\text{-}m}}_{= q} + r $$

$$ \mathbb{K} \sqsupset \diagdown \text{ field ext } \Longrightarrow {}^{X}\triangle_{\bullet}\mathbb{K} \xleftarrow[\text{hom}]{{}_{\varkappa}\daleth | \daleth} {}^{X}\triangle_{\bullet}\mathbb{k} $$

$$ {}^{X}\daleth = X^{i}{}_{i}\daleth \Longrightarrow {}^{Y}\daleth_{\varkappa} = Y^{i}{}_{i}\daleth_{\varkappa} $$

$$ a \in \mathbb{k} \Longrightarrow {}^{\varkappa a}_{\varkappa}\daleth = \varkappa^{a}\daleth $$

$$ \text{RHS } = \varkappa\left(a^{i}{}_{i}\daleth\right) = (\varkappa a)^{i}\left(\varkappa_{i}\daleth\right) = \text{ LHS} $$