

$$\mathfrak{h}_{\mathbb{R}} \xrightarrow[u:\mathbb{R}]{u:v} \mathbb{R}$$

$${}^{x+iy}\mathfrak{V} = {}^{x:y}u + i {}^{x:y}v$$

$$\frac{\partial (u:v)}{\partial (x:y)} = \frac{\frac{\partial u}{\partial x} \Big| \frac{\partial v}{\partial x}}{\frac{\partial u}{\partial y} \Big| \frac{\partial v}{\partial y}}$$

$$\mathfrak{V} \text{ diff in } c = a + ib \Leftrightarrow \overline{z - c} \leqslant \delta \curvearrowright \overline{{}^z\mathfrak{V} - {}^c\mathfrak{V} - (z - c) \partial_z^c \mathfrak{V} - (z^* - c^*) \partial_{z^*}^c \mathfrak{V}} \leqslant \varepsilon \overline{{}^n z - c^n}$$

$$(z - c) \partial_z \mathfrak{V} + (z^* - c^*) \partial_{z^*} \mathfrak{V} = ((x - a) + i(y - b)) \partial_z \mathfrak{V} + ((x - a) - i(y - b)) \partial_{z^*} \mathfrak{V}$$

$$= (x - a) \left(\partial_z \mathfrak{V} + \partial_{z^*} \mathfrak{V} \right) + i(y - b) \left(\partial_z \mathfrak{V} - \partial_{z^*} \mathfrak{V} \right) = (x - a) (\partial_x u + i \partial_x v) + (y - b) (\partial_y u + i \partial_y v)$$

$$\Rightarrow {}^z\mathfrak{V} - {}^c\mathfrak{V} - (z - c) {}^c\partial_z \mathfrak{V} - (z^* - c^*) {}^c\partial_{z^*} \mathfrak{V}$$

$$= {}^{x:y}u - {}^{a:b}u - (x - a) {}^{a:b}\partial_x u - (y - b) {}^{a:b}\partial_y u + i \left({}^{x:y}v - {}^{a:b}v - (x - a) {}^{a:b}\partial_x v - (y - b) {}^{a:b}\partial_y v \right)$$

$$\mathfrak{V} \text{ diff} \Leftrightarrow u:v \text{ diff} \Leftrightarrow \overline{{}^n x - a:y - b^n} \leqslant \delta \curvearrowright$$

$$\overline{{}^{x:y}u - {}^{a:b}u - (x - a) {}^{a:b}\partial_x u - (y - b) {}^{a:b}\partial_y u} \leqslant \frac{\varepsilon}{2} \overline{{}^n x - a:y - b^n} \geqslant \overline{{}^{x:y}v - {}^{a:b}v - (x - a) {}^{a:b}\partial_x v - (y - b) {}^{a:b}\partial_y v}$$