

$$\partial_z \mathfrak{V} = \frac{1}{2} (\partial_x \mathfrak{V} - i \partial_y \mathfrak{V}) = \frac{1}{2} (\partial_x u + \partial_y v) + \frac{i}{2} (\partial_x v - \partial_y u)$$

$$\partial_{\bar{z}} \mathfrak{V} = \frac{1}{2} (\partial_x \mathfrak{V} + i \partial_y \mathfrak{V}) = \frac{1}{2} (\partial_x u - \partial_y v) + \frac{i}{2} (\partial_x v + \partial_y u)$$

$$\partial_z \mathfrak{V} + \partial_{\bar{z}} \mathfrak{V} = \partial_x \mathfrak{V}$$

$$\partial_z \mathfrak{V} - \partial_{\bar{z}} \mathfrak{V} = -i \partial_y \mathfrak{V}$$

$$\frac{\partial \left(\mathfrak{V} : \bar{\mathfrak{V}} \right)}{\partial \left(z : \bar{z} \right)} = \frac{\frac{\partial \mathfrak{V}}{\partial z} \left| \frac{\partial \bar{\mathfrak{V}}}{\partial \bar{z}} \right.}{\frac{\partial \mathfrak{V}}{\partial \bar{z}} \left| \frac{\partial \bar{\mathfrak{V}}}{\partial z} \right.}$$

$$\frac{\partial \left(\mathfrak{V} : \bar{\mathfrak{V}} \right)}{\partial \left(z : \bar{z} \right)} = \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\partial (u:v)}{\partial (x:y)} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.}$$

$$\frac{\frac{\partial_z \mathfrak{V}}{\partial_{\bar{z}} \mathfrak{V}} \left| \frac{\partial_z \bar{\mathfrak{V}}}{\partial_{\bar{z}} \bar{\mathfrak{V}}} \right.}{\frac{\partial_z \mathfrak{V}}{\partial_{\bar{z}} \mathfrak{V}} \left| \frac{\partial_z \bar{\mathfrak{V}}}{\partial_{\bar{z}} \bar{\mathfrak{V}}} \right.} = \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\frac{\partial_x u}{\partial_y u} \left| \frac{\partial_x v}{\partial_y v} \right.}{\frac{\partial_x u}{\partial_y u} \left| \frac{\partial_x v}{\partial_y v} \right.} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} = \frac{1 \left| \begin{smallmatrix} -i \\ i \end{smallmatrix} \right.}{1 \left| i \right.} / \sqrt{2} \frac{\frac{\partial_x u}{\partial_y u} \left| \frac{\partial_x v}{\partial_y v} \right.}{\frac{\partial_x u}{\partial_y u} \left| \frac{\partial_x v}{\partial_y v} \right.} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} / \sqrt{2}$$

unitary

$$\frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\frac{\partial_z \mathfrak{V}}{\partial_{\bar{z}} \mathfrak{V}} \left| \frac{\partial_z \bar{\mathfrak{V}}}{\partial_{\bar{z}} \bar{\mathfrak{V}}} \right.}{\frac{\partial_z \mathfrak{V}}{\partial_{\bar{z}} \mathfrak{V}} \left| \frac{\partial_z \bar{\mathfrak{V}}}{\partial_{\bar{z}} \bar{\mathfrak{V}}} \right.} = \frac{\frac{\partial_z \mathfrak{V} + \partial_{\bar{z}} \mathfrak{V}}{i \partial_z \mathfrak{V} - i \partial_{\bar{z}} \mathfrak{V}} \left| \frac{\partial_z \bar{\mathfrak{V}} + \partial_{\bar{z}} \bar{\mathfrak{V}}}{i \partial_z \bar{\mathfrak{V}} - i \partial_{\bar{z}} \bar{\mathfrak{V}}} \right.}{\frac{\partial_x \mathfrak{V}}{\partial_y \mathfrak{V}} \left| \frac{\partial_x \bar{\mathfrak{V}}}{\partial_y \bar{\mathfrak{V}}} \right.} = \frac{\partial_x \mathfrak{V}}{\partial_y \mathfrak{V}} \left| \frac{\partial_x \bar{\mathfrak{V}}}{\partial_y \bar{\mathfrak{V}}} \right.$$

$$= \frac{\frac{\partial_x u + i \partial_x v}{\partial_y u + i \partial_y v} \left| \frac{\partial_x u - i \partial_x v}{\partial_y u - i \partial_y v} \right.}{\frac{\partial_x u}{\partial_y u} \left| \frac{\partial_x v}{\partial_y v} \right.} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.}$$

$$\partial_z \overline{\mathfrak{V} \mathfrak{K}} = \overline{\partial_z \mathfrak{V}} \overline{\partial_w \mathfrak{K}} + \overline{\partial_z \bar{\mathfrak{V}}} \overline{\partial_{\bar{w}} \mathfrak{K}}$$

$$\partial_{\bar{z}} \overline{\mathfrak{V} \mathfrak{K}} = \overline{\partial_{\bar{z}} \mathfrak{V}} \overline{\partial_w \mathfrak{K}} + \overline{\partial_{\bar{z}} \bar{\mathfrak{V}}} \overline{\partial_{\bar{w}} \mathfrak{K}}$$

$$^{x+iy} \mathfrak{V} = {}^{x:y} u + i {}^{x:y} v : {}^{u+iv} \mathfrak{K} = {}^{u:v} p + i {}^{u:v} q : {}^{x+iy} \overline{\mathfrak{V} \mathfrak{K}} = {}^{x:y} p + i {}^{x:y} q$$

$$\Rightarrow \frac{\partial \left(\mathfrak{V} : \bar{\mathfrak{V}} \right)}{\partial \left(z : \bar{z} \right)} \frac{\partial \left(\mathfrak{K} : \bar{\mathfrak{K}} \right)}{\partial \left(w : \bar{w} \right)} = \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\partial (u:v)}{\partial (x:y)} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\partial (p:q)}{\partial (u:v)} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.}$$

$$= \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\partial (u:v)}{\partial (x:y)} \frac{\partial (p:q)}{\partial (u:v)} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} = \frac{1 \left| \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right.}{i \left| -i \right.} \frac{\partial (p:q)}{\partial (x:y)} \frac{1 \left| \begin{smallmatrix} 1 \\ -1 \end{smallmatrix} \right.}{i \left| -i \right.} = \frac{\partial \left(\mathfrak{V} \mathfrak{K} : \bar{\mathfrak{V}} \bar{\mathfrak{K}} \right)}{\partial \left(z : \bar{z} \right)}$$