$$h^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\,\eta_{\mu\nu} = e^\alpha_a\,\eta^{ab}\,e^\beta_b\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\,\eta_{\mu\nu} = \underbrace{e^\alpha_a\partial_\alpha X^\mu}\,\eta^{ab}\,\eta_{\mu\nu}\,\underbrace{e^\beta_b\partial_\beta X^\nu}$$

$$-i\,\bar\psi^\mu\,\varrho^\alpha\,\partial_\alpha\,\psi^\nu\,\eta_{\mu\nu} = -i\,\bar\psi^\mu\,e^\alpha_a\,\varrho^a\,\partial_\alpha\,\psi^\nu\,\eta_{\mu\nu} = -i\,\bar\psi^\mu\,\eta_{\mu\nu}\,\varrho^a\,\underbrace{e^\alpha_a\partial_\alpha\psi^\nu}$$

$$\bar\chi_\alpha\,\varrho^\beta\,\varrho^\alpha\,\psi^\mu\,\partial_\beta X^\nu\,\eta_{\mu\nu} = \bar\chi_\alpha\,e^\beta_b\,\varrho^b\,e^\alpha_a\,\varrho^a\,\psi^\mu\,\partial_\beta X^\nu\,\eta_{\mu\nu} = \underbrace{\bar\chi_\alpha e^\alpha_a}\,\varrho^b\,\varrho^a\,\psi^\mu\,\eta_{\mu\nu}\,\underbrace{e^\beta_b\partial_\beta X^\nu}$$

$$\bar\psi^\mu\,\eta_{\mu\nu}\,\bar\psi^\nu\,\bar\chi_\alpha\,\varrho^\beta\,\varrho^\alpha\,\chi_\beta = \bar\psi^\mu\,\eta_{\mu\nu}\,\bar\psi^\nu\,\bar\chi_\alpha\,e^\beta_b\,\varrho^b\,e^\alpha_a\,\varrho^a\,\chi_\beta = \bar\psi^\mu\,\eta_{\mu\nu}\,\bar\psi^\nu\,\underbrace{\bar\chi_\alpha e^\alpha_a}\,\varrho^b\,\varrho^a\,\underbrace{e^\beta_b\chi_\beta}$$

$$\omega^A = \omega^a\,|\,\omega^\alpha = dx^a + id\vartheta\,\varrho^a\,\bar\vartheta\,|d\vartheta^\alpha = dx^\mu|d\vartheta^\mu\,\begin{array}{c|c} e^a_m & e^\alpha_m \\ \hline e^a_\mu & e^\alpha_\mu \end{array} = dx^\mu|d\vartheta^\mu\,\begin{array}{c|c} e^a_m & 0 \\ \hline e^a_\mu & \delta^\alpha_\mu \end{array}$$

$$\mathcal{D}_A = \begin{bmatrix}\mathcal{D}_a\\ \mathcal{D}_\alpha\end{bmatrix} = \overset{-1}{E}{}^M_A\,\partial_M = \begin{array}{c|c} \overset{-1}{e}{}^m_a & 0 \\ \hline -e^a_\alpha\overset{-1}{e}{}^m_a & \delta^\mu_\alpha \end{array}\,\begin{bmatrix}\partial_m\\ \partial_\mu\end{bmatrix} = \begin{bmatrix}\overset{-1}{e}{}^m_a\partial_m\\ \partial_\mu - e^a_\alpha\overset{-1}{e}{}^m_a\partial_m\end{bmatrix}$$

$$\mathcal{D}_a = \overset{-1}{e}{}^m_a\,\partial_m$$

$$\mathcal{D}_\alpha = \partial_\alpha - e^a_\alpha\,\overset{-1}{e}{}^m_a\,\partial_m$$

$$dx^m\,E^a_m + d\vartheta^\mu\,E^a_\mu = dx^a + id\vartheta\,\varrho^a\,\bar\vartheta \Longrightarrow \begin{cases} dx^m E^a_m = dx^a \\ d\vartheta^\mu E^a_\mu = id\vartheta\varrho^a\bar\vartheta \end{cases}$$

$$dx^m\,E^\alpha_m + d\vartheta^\mu\,E^\alpha_\mu = d\vartheta^\alpha \Longrightarrow \begin{cases} E^\alpha_m = 0 \\ E^\alpha_\mu = \partial^\alpha_\mu \end{cases}$$

$$\text{guess}\ \left(\varrho^a\vartheta\right)_A = \chi^\mu_A\,e^a_\mu$$

$$\mathcal{D}_A = \partial_A - \chi^\mu_A\,\partial_\mu - i\left(\varrho^a\vartheta\right)_A\,e^\mu_a\,\partial_\mu$$

$$= \partial_A - \left(\chi^\mu_A + i(\varrho^a\vartheta)_A e^\mu_a\right)\,\partial_\mu$$

$$= \partial_A - \left(\chi^\mu_A e^a_\mu + i(\varrho^a\vartheta)_A\right)\,e^\mu_a\,\partial_\mu$$