

$$\mathbb{H}_0^d \subset \mathbb{H}^{1:d} \text{ spacetime}$$

$\mathbb{H}_{\infty}^{\mathbb{H}}$ fields

$$\underline{\mathbb{H} \times \Psi}^\varphi = \underline{\mathbb{H} \times \Psi}_{\varphi_h}$$

$$\frac{\mathbb{H}: \varphi: \Phi}{\mathbb{H} \in \mathbb{H}: \quad \varphi \in \mathbb{H} \times \Psi: \quad \mathbb{H} \times \mathbb{H}} \xrightarrow[\text{lin}]{\Phi} \underline{\mathbb{H} \times \Psi}_\varphi \xrightarrow[\text{Lag}]{\mathcal{L}} \mathbb{R}$$

$$\mathbb{H}: \varphi: \Phi \mapsto {}_{\mathbb{H}}^{\varphi: \Phi} \mathcal{L}$$

$$\underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow{\mathbb{R}} \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[p:q]{} \mathbb{R}$$

$$\mathcal{L} \in \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow{\mathbb{R}} \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[0:1+d]{} \mathbb{R} \text{ density}$$

$${}^{\mathbb{H}} \mathcal{L}_\varphi = {}^{\mathbb{H} \varphi: \mathbb{H}} \underline{\mathcal{L}}_h$$

$$\begin{array}{ccccc} \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} & \xrightarrow[p:q+1]{} & d & \xleftarrow{\quad} & \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[p:q]{} \\ & & D & & \\ & \searrow & & & \downarrow \partial \\ & & \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} & \xrightarrow[\mathbb{R}]{} & \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[p+1:q]{} \mathbb{R} \end{array}$$

$$\underline{\mathbb{H} \times \Psi}^\varphi \xrightarrow[\text{lin}]{D\mathcal{L}} \mathbb{H} \times \mathbb{R}$$

$$D\mathcal{L} = \partial \mathcal{L} + d\gamma \in \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow{\mathbb{R}} \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[1:1+d]{} \mathbb{R}$$

$$\gamma \in \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow{\mathbb{R}} \underbrace{\mathbb{H}_{\infty}^{\mathbb{H}} \times \mathbb{H}}_{\infty} \xrightarrow[1:d]{} \mathbb{R}$$

$$\Omega = \partial \gamma \in \overbrace{\Delta_\infty^{\frac{h}{2}} \times \Delta_\infty^{\frac{h}{2}}}^{\frac{h}{2} \times \frac{h}{2}} \underbrace{\Delta_\infty^{\frac{h}{2}} \times \Delta_\infty^{\frac{h}{2}}}_{\frac{h}{2} \times \frac{h}{2}}$$

$$\omega = \int^{h_0} \Omega \in \overbrace{\Delta_\infty^{\frac{h}{2}}}^{\frac{h}{2}} \underbrace{\Delta_\infty^{\frac{h}{2}}}_{\frac{h}{2}} \text{ sympl}$$

$$T \times_{\overbrace{\Delta_\infty^{\frac{h}{2}}}^{\frac{h}{2}}} \frac{t^i : q_\alpha^j : |\alpha| \leq k}{\text{coord}} \mathbb{R}$$

$${}^\psi q_\alpha^j=\partial_\alpha\left({}^\psi\rtimes q^j\right)$$

$$dt^i=Dt^i$$

$$\delta t^i=0$$

$$\delta q_\alpha^j=\delta q_\alpha^j$$

$$Dq_\alpha^j=\begin{cases} dt^iq_\alpha^j+\varepsilon_i & |\alpha|< k \\ 0 & |\alpha|=k \end{cases}$$

$$DF=dt^i\left(\frac{\partial F}{\partial t^i}+\frac{\partial F}{\partial q_\alpha^j}q_\alpha^j\right)$$

$$\delta F=\frac{\partial F}{\partial q_\alpha^j}\delta q_\alpha^j$$

$$\begin{aligned} dF &= \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} dq_\alpha^j = \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} \underbrace{Dq_\alpha^j + \delta q_\alpha^j}_{=DF} = \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} \underbrace{dt^i q_\alpha^j + \delta q_\alpha^j}_{=\delta F} \\ &= \underbrace{dt^i \frac{\partial F}{\partial t^i} + \frac{\partial F}{\partial q_\alpha^j} q_\alpha^j}_{=DF} + \underbrace{\frac{\partial F}{\partial q_\alpha^j} \delta q_\alpha^j}_{=\delta F} \end{aligned}$$

$$\delta \underline{\mathcal{L}} dt = {}_{\text{source}} E + D \underline{M}_{0:1} \Rightarrow M = - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j$$

$$E = \underbrace{\frac{\partial \mathcal{L}}{\partial q^j} - \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j}_{\delta q^j} \wedge dt$$

$$\begin{aligned}
D \left(\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j \right) &= D \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \star \delta q^j + \frac{\partial \mathcal{L}}{\partial \dot{q}^j} D \delta q^j = \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \star \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta D q^j \\
&= \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \star \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta dt \star \dot{q}^j = \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \star \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \star \delta \dot{q}^j \\
\delta \underline{\mathcal{L}} dt &= d \underline{\mathcal{L}} dt = \underline{\mathcal{L}} \star dt = \left(\frac{\partial \mathcal{L}}{\partial t} dt + \frac{\partial \mathcal{L}}{\partial q^j} dq^j + \frac{\partial \mathcal{L}}{\partial \dot{q}^j} d\dot{q}^j \right) \star dt \\
&= \overline{\frac{\partial \mathcal{L}}{\partial q^j} Dq^j + \delta q^j} + \overline{\frac{\partial \mathcal{L}}{\partial \dot{q}^j} D\dot{q}^j + \delta \dot{q}^j} \star dt = \overline{\frac{\partial \mathcal{L}}{\partial q^j} dt \dot{q}^j + \delta q^j} + \overline{\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta \dot{q}^j} \star dt \\
&\Rightarrow \delta \underline{\mathcal{L}} dt + D \left(\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j \right) = \underbrace{\frac{\partial \mathcal{L}}{\partial q^j} - \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j}_{\delta q^j} \wedge dt
\end{aligned}$$