$$\mathtt{L}_0 \xrightarrow[\text{in}]{\partial_0} \mathtt{L}_1 \xrightarrow[\text{ex}]{\partial_1} \cdots \xrightarrow[\text{ex}]{\partial_{n-2}} \mathtt{L}_{n-1} \xrightarrow[\text{ex}]{\partial_{n-1}} \mathtt{L}_n \xrightarrow[\text{on}]{\partial_n} \mathtt{L} \Longrightarrow \dim \mathtt{L} = \sum_{0 \leqslant i \leqslant n} {}^{n-i}{-1} \dim \mathtt{L}_i$$

$$\mathtt{L}_0 \xrightarrow[\text{in}]{\partial_0} \mathtt{L}_1 \xrightarrow[\text{ex}]{\partial_1} \cdots \xrightarrow[\text{ex}]{\partial_{n-2}} \mathtt{L}_{n-1} \xrightarrow[\text{on}]{\partial_{n-1}} \ker \partial_n \underset{\text{ind}}{\Longrightarrow} \dim \ker \partial_n = \sum_{0 \leqslant i < n} {}^{n-1-i}{-1} \dim \mathtt{L}_i$$

$$\begin{aligned} \dim \mathtt{L} &= \dim \mathtt{L}_n - \dim \ker \partial_n = \dim \mathtt{L}_n - \sum_{0 \leqslant i < n} {}^{n-1-i}{-1} \dim \mathtt{L}_i \\ &= \dim \mathtt{L}_n + \sum_{0 \leqslant i < n} {}^{n-i}{-1} \dim \mathtt{L}_i = \sum_{0 \leqslant i \leqslant n} {}^{n-i}{-1} \dim \mathtt{L}_i \end{aligned}$$