



$${}^n \mathbb{C}_n^{\mathbb{C}} \ni \frac{a \mid b}{c \mid d}$$

$${}^n \mathbb{R}_n^{\mathbb{C}} \ni \frac{a \mid b}{c \mid d}$$

$${}^n \mathbb{H}_n^{\mathbb{C}} \ni \frac{a \mid b}{-\bar{b} \mid \bar{a}}$$

$$\frac{\bar{a} \mid \bar{b}}{\bar{c} \mid \bar{d}} = J \frac{a \mid b}{c \mid d} \quad J^{-1} = \frac{d \mid -c}{-b \mid a} \Leftrightarrow \begin{cases} a = \bar{d} \\ b = -\bar{c} \end{cases}$$

$${}^n \mathbb{C}_n^{\mathbb{C}} \ni \frac{a \mid b}{-b \mid a}$$

$$\frac{a \mid b}{c \mid d} = J \frac{a \mid b}{c \mid d} \quad J^{-1} = \frac{d \mid -c}{-b \mid a} \Leftrightarrow \begin{cases} a = d \\ b = -c \end{cases}$$

$$\frac{a \mid b}{-\bar{b} \mid \bar{a}} = J \frac{a \mid b}{-\bar{b} \mid \bar{a}} \quad J^{-1} = \frac{\bar{a} \mid \bar{b}}{-b \mid a} \Leftrightarrow \begin{cases} a = \bar{a} \\ b = \bar{b} \end{cases}$$

$$\begin{array}{c} g \in \left\{ \begin{smallmatrix} {}^n\mathbb{H}_n^{\mathbb{C}} \\ {}^n\mathbb{C}_n^{\mathbb{C}} \\ {}^n\mathbb{R}_n^{\mathbb{C}} \\ {}^2\mathbb{R}_n \end{smallmatrix} \right. \\ \left. \frac{{}^n\mathbb{C}_n^{\mathbb{C}}}{{}^n\mathbb{R}_n^{\mathbb{C}}} \right\} \\ gJ = Jg \end{array}$$

$$\begin{cases} gJ = J\bar{g} \\ gJ = Jg \end{cases} \Rightarrow g = \bar{g}$$

$$J \frac{a}{-b} \begin{array}{c|c} b & -1 \\ \hline \bar{a} & a \end{array} = \frac{\bar{a}}{-b} \begin{array}{c|c} \bar{b} & -1 \\ \hline a & \bar{a} \end{array} = \frac{a}{-b} \begin{array}{c|c} b & -1 \\ \hline \bar{a} & a \end{array} \Leftrightarrow \begin{cases} a = \bar{a} \\ b = \bar{b} \end{cases}$$

$$\begin{cases} gJ = Jg \\ g = \bar{g} \end{cases} \Rightarrow gJ = J\bar{g}$$

$$J \frac{a}{c} \begin{array}{c|c} b & -1 \\ \hline d & a \end{array} = \frac{d}{-b} \begin{array}{c|c} -c & -1 \\ \hline a & d \end{array} = \frac{a}{c} \begin{array}{c|c} b & -1 \\ \hline d & a \end{array} \Leftrightarrow \begin{cases} a = d \\ b = -c \end{cases}$$

$$\overbrace{J - z}^{-1} \underbrace{J + z} \in {}^n\mathbb{R}_n^{\mathbb{C}} \Leftrightarrow z \in {}^n\mathbb{R}_n^{\mathbb{C}}$$

$$\frac{\begin{cases} {}^n\mathbb{H}_n^{\mathbb{C}} \\ {}^n\mathbb{C}_n^{\mathbb{C}} \\ {}^n\mathbb{R}_n^{\mathbb{C}} \\ {}^2\mathbb{R}_n \end{cases}}{ {}^n\mathbb{C}_n^{\mathbb{C}}} = \frac{g}{gJ \cdot Jg = -1}$$

$$Jz = -zJ \Leftrightarrow z = \frac{a}{b} \begin{array}{c|c} b & -1 \\ \hline -a & a \end{array}$$

$$\frac{a}{b} \begin{array}{c|c} b & -1 \\ \hline -a & a \end{array} J = \frac{b}{-a} \begin{array}{c|c} a & -1 \\ \hline b & a \end{array} \in {}^n\mathbb{C}_n^{\mathbb{C}}$$

$$\frac{a}{0} \left| \begin{array}{c} 0 \\ -a \end{array} \right. \in \xrightarrow{\quad} \frac{{}^n\mathbb{H}_n^{\mathbb{C}}}{\frac{{}^n\mathbb{C}_n^{\mathbb{C}}}{\mathbb{R}}} \ni \frac{\overbrace{1-a^2}^{-1} \overbrace{1+a^2}}{2a\underbrace{1-a^2}_{-1}} \left| \begin{array}{c} 2a \overbrace{1-a^2}^{-1} \\ \underbrace{1+a^2 \overbrace{1-a^2}_{-1}} \end{array} \right.$$

$${}^n\mathbb{C}_n \xrightarrow{\overbrace{\mathcal{J}-z}^{-1} \overbrace{\mathcal{J}+z}} {}_2^n\mathbb{C}_n$$

$$\frac{a}{b} \left| \begin{array}{c} b \\ -a \end{array} \right. \in {}_2^n\mathbb{C}_n J \xrightarrow{\quad} \frac{{}^n\mathbb{R}_n^{\mathbb{C}}}{\frac{{}^n\mathbb{C}_n^{\mathbb{C}}}{\mathbb{R}}} \ni \frac{\overbrace{1+b}^{-1} \overbrace{1-b}}{0} \left| \begin{array}{c} 0 \\ \underbrace{1-b \overbrace{1+b}_{-1}} \end{array} \right.$$

$$\begin{array}{ccc} & {}^n\mathbb{C}_n^{\mathbb{C}} & \\ \nearrow & & \searrow \\ \frac{0}{-i} \left| \begin{array}{c} i \\ 0 \end{array} \right. & & \frac{1}{0} \left| \begin{array}{c} 0 \\ -1 \end{array} \right. \\ & & \\ \frac{1}{-i} \left| \begin{array}{c} i \\ -1 \end{array} \right. {}^n\mathbb{H}_n^{\mathbb{C}} \frac{1}{-i} \left| \begin{array}{c} i \\ -1 \end{array} \right. & & {}^n\mathbb{R}_n^{\mathbb{C}} \\ & \swarrow & \nearrow \\ \frac{1}{0} \left| \begin{array}{c} 0 \\ -1 \end{array} \right. & & \frac{0}{-i} \left| \begin{array}{c} i \\ 0 \end{array} \right. \\ & & {}^n\mathbb{C}_n^{\mathbb{C}} \end{array}$$

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