power integrals

$$\int_{dx} x^a = \frac{x^{a+1}}{a+1}$$

$$\int_{dx} 1 = x$$

$$\int_{dx} x = \frac{x^2}{2}$$

$$\int_{dx} \frac{1}{x^2} = -\frac{1}{x}$$

$$\int_{dx} \frac{1}{\sqrt{x}} = 2\sqrt{x}$$

$$\int_{dx} \sqrt{x} = \frac{2}{3}\,x^{3/2}$$

$$\int_{dx} \frac{1}{x} = \log x$$

$$\int_{dx} 3\,x^5 + 7\,x^4 + 3\,x^2 + 2x = \frac{1}{2}\,x^6 + \frac{7}{5}\,x^5 + x^3 + x^2$$

$$\int_{dx} \left(x^2 - 1\right)^3 = \frac{1}{7}\,x^7 - \frac{3}{5}\,x^5 + x^3 - x$$

$$\int_{dx} \left(x^2 + x + 1\right)^2 = \frac{1}{5}\,x^5 + \frac{1}{2}\,x^4 + x^3 + x^2 + x$$

$$\int_{dx} \frac{x^3 + x^2 + x + 1}{x^2} = \frac{1}{2}\,x^2 + x - \frac{1}{x} + \log x$$

$$\int_{dx} \frac{2x^3 + 8x + 5}{x} = \frac{2}{3}\,x^3 + 8x + 5\log x$$

$$\int\limits_{dx} \frac{x^{1/5}+x^{1/2}}{x} = 2\sqrt{x} + 5\,x^{1/5}$$

$$\int\limits_{dx} \frac{3x^{2/3}+2x^{1/3}+x^{1/5}}{x^{1/4}} = \frac{36}{17}\,x^{17/12} + \frac{24}{13}\,x^{13/12} + \frac{20}{19}\,x^{19/20}$$

$$\int\limits_{dx} \frac{x+x^{1/4}}{x^{1/3}} = \frac{3}{5}\,x^{5/3} + \frac{12}{11}\,x^{11/12}$$

$$\int\limits_{dx} \frac{1-(2-\sqrt{x})}{x^{1/3}} = \frac{24}{7}\,x^{7/6} - \frac{3}{5}\,x^{5/3} - \frac{9}{2}\,x^{2/3}$$

$$\int\limits_{dx} \frac{x^{1/4}+x^{2/7}}{x^{3/2}} = -4\,x^{-1/4} - \frac{14}{3}\,x^{-3/14}$$

$$\int\limits_{dx} \frac{\left(\sqrt{x}-3\right)^2\left(\sqrt{x}+3\right)^2}{x^2\sqrt{x}} = 2\sqrt{x} + \frac{36}{\sqrt{x}} - \frac{54}{x\sqrt{x}}$$

$$\int\limits_{dx} \left(2x+5\right)^{1/3} = \frac{3}{8}\left(2x+5\right)^{4/3}$$

$$\int\limits_{dx} \sqrt{3x+5} = \frac{2}{9}\left(3x+5\right)^{3/2}$$

$$\int\limits_{dx} x\sqrt{x^2+7} = \frac{1}{3}\left(x^2+7\right)^{3/2}$$

$$\int\limits_{dx} x^2\left(2-5x^3\right)^{1/4} = -\frac{4}{75}\left(2-5x\right)^{5/4}$$

$$\int\limits_{dx} x\sqrt{2x+3} = \frac{1}{10}\left(2x+3\right)^{5/2} - \frac{1}{2}\left(2x+3\right)^{3/2}$$

$$\int\limits_{dx} \frac{\left(3-5\sqrt{x}\right)^4}{\sqrt{x}} = -\frac{2}{25}\left(3-5\sqrt{x}\right)^5$$

$$\int\limits_{dx} x\left(2x-3\right)^{1/5} = \frac{5}{44}\left(2x-3\right)^{11/5} + \frac{5}{8}\left(2x-3\right)^{6/5}$$

$$\int_{dx} x^2\sqrt{3x-1} = \frac{2}{189}(3x-1)^{7/2} + \frac{4}{135}(3x-1)^{5/2} + \frac{2}{81}(3x-1)^{3/2}$$

$$\int_{dx} x^2\left(x^3+2\right)^{1/3} = \frac{1}{4}\left(x^3+2\right)^{4/3}$$

exp integrals

$$\int \exp x = \exp x$$

$$\int a^x = \frac{a^x}{\log a}$$

$$\text{Sub} \int_{dx} \frac{\exp(1/x)}{x^2}$$

$$\text{part} \int_{dx} \begin{cases} x\exp x = x\exp x - \exp x \\ x\underbrace{\exp(-3x)}_{=g'} = x\frac{\exp(-3x)}{-3} - \int_{dx} \frac{\exp(-3x)}{-3} \models \frac{x\exp(-3x)}{-3} - \frac{\exp(-3x)}{9} \end{cases}$$

$$\int_{dx} \frac{1}{\exp x + \exp(-x)} = \arctan(\exp x): \quad \int_{dx} \frac{2}{\exp(2x)+\exp(-2x)} = \arctan(\exp(2x))$$

$$\int_{dx} \exp x\sqrt{1+\exp x} = \frac{2}{3}(1+\exp x)^{3/2}$$

$$\int_{dx} \frac{\exp x}{\exp(2x)+\exp x + 1} = \frac{2}{\sqrt{3}}\arctan\left((1+2\exp x)/\sqrt{3}\right)$$

$$\int_{dx} \frac{2}{\sqrt{\exp(2x)-\exp x+1}} = \log\left(\frac{2\sqrt{\exp(2x)-\exp x+1}+\mathfrak{e}^x-2}{2\sqrt{\exp(2x)-\exp x+1}-\mathfrak{e}^x+2}\right)$$

$$\int_{dx} \frac{2\sqrt{7}}{\sqrt{7-3\exp(2x)}} = \log\left(\frac{\sqrt{7}-\sqrt{7-3\exp(2x)}}{\sqrt{7}+\sqrt{7-3\exp(2x)}}\right)$$

$$\int_{dx} \frac{5}{\exp x+5} = \log\left(\frac{\exp x}{\exp x+5}\right)$$

$$\int\limits_{dx} \frac{\sqrt{\exp(-x)+1}}{\exp x} = -\frac{2}{3}\left(\exp(-x)+1\right)^{3/2}$$

$$\int\limits_{dx} x\exp(1-x) = -(x+1)\exp(1-x)$$

$$\int\limits_{dx} x^3\exp\left(-x^2\right) = -\frac{x^2+1}{2}\exp\left(-x^2\right)$$

rational integrals substitution

$$\int\limits_{dx} x\left(x^2-1\right)^3 = \frac{1}{8}\left(x^2-1\right)^4$$

$$\int\limits_{dx} x^2\left(x^3-1\right)^{20} = \frac{1}{63}\left(x^3-1\right)^{21}$$

$$\int\limits_{dx} \frac{4x^3}{x^4+1} = \log\left(x^4+1\right)$$

$$\int\limits_{dx} \frac{3}{3x+7} = \log(3x+7)$$

$$\int\limits_{dx} \frac{4x+3}{2x^2+3x+1} = \log\left(2x^2+3x+1\right)$$

$$\int\limits_{dx} \frac{3x^2-1}{x^3-x+1} = \log\left(x^3-x+1\right)$$

$$\int\limits_{dx} \frac{2x-1}{x^2-x+1} = \log\left(x^2-x+1\right)$$

$$\int\limits_{dx} \frac{3x}{\left(x^2+7\right)^3} = -\frac{3}{4}\left(x^2+7\right)^{-2}$$

rational integrals linear

$$\frac{1}{(x-a)^n}$$

$$\int \begin{cases} \frac{1}{x-a} \\ \frac{1}{(x-a)^n} \end{cases} = \begin{cases} \log(x-a) \\ \frac{-1}{(n-1)(x-a)^{n-1}} \end{cases}$$

$$\int_{dx} \frac{1}{x^2-5x+6} = \log\left(\frac{x-3}{x-2}\right)$$

$$\int_{dx} \frac{3x+4}{x^2+5x+6} = 5\log(x+3) - 2\log(x+2)$$

$$\int_{dx} \frac{x^3-1}{x-1} = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

$$\int_{dx} \frac{3x^3+5x+1}{x+1} = x^3 - \frac{3}{2}x^2 + 8x - 7\log(x+1)$$

$$\int_{dx} \frac{x+1}{2x+3} = \frac{1}{2}x - \frac{1}{4}\log(2x+3)$$

$$\int_{dx} \frac{2x+1}{x^2+x+7} = \log\left(x^2+x+7\right)$$

$$\int_{dx} \frac{40x^4+42x^2}{4x^5+7x^3+1} = 2\log\left(4x^5+7x^3+1\right)$$

$$\int_{dx} \frac{7x+5}{x^2+8x+12} = \frac{37}{4}\log(x+6) - \frac{9}{4}\log(x+2)$$

$$\int_{dx} \frac{5x+8}{x^2+6x-7} = \frac{27}{8}\log(x+7) + \frac{13}{8}\log(x-1)$$

$$\int_{dx} \frac{x^3}{x^2+3} = \frac{1}{2}x^2 - \frac{3}{2}\log\left(x^2+3\right)$$

$$\int_{dx} \frac{2x}{5x^2+7} = \frac{1}{5}\log\left(5x^2+7\right)$$

$$\int_{dx} \frac{2x}{x^2-x-1} = \log\left(x^2-x-1\right) - \frac{1}{\sqrt{5}}\log\left(\frac{2x-1+\sqrt{5}}{2x-1-\sqrt{5}}\right)$$

$$\int_{dx} \frac{3x+7}{x^2+4x+4} = -\left(x-3\right)^{-1}$$

$$\int_{dx} \frac{x+4}{9x^2+6x+1} = -\left(x+2\right)^{-1} + 3\log\left(x+2\right)$$

$$\int_{dx} \frac{3x+7}{x^2-4x+4} = -\frac{11}{9}\left(3x+1\right)^{-1} + \frac{1}{9}\log\left(3x+1\right)$$

$$\int_{dx} \frac{5x+1}{4x^2+4x+1} = -13\left(x-2\right)^{-1} + 3\log\left(x-2\right)$$

$$\int_{dx} \frac{7x+5}{\left(3x-1\right)^2} = \frac{3}{4}\left(2x+1\right)^{-1} + \frac{5}{4}\log\left(2x+1\right)$$

$$\int_{dx} \frac{2x-5}{\left(x-1\right)^3} = -\frac{22}{9}\left(3x-1\right)^{-1} + \frac{7}{9}\log\left(3x-1\right)$$

$$\int_{dx} \frac{8x^2+5x+7}{\left(2x+3\right)^3} = \log\left(2x+3\right) + \frac{19}{4}\left(2x+3\right)^{-1} - \frac{35}{8}\left(2x+3\right)^{-2}$$

$$\int_{dx} \frac{x^2+5x+7}{x^2-5x} = x - \frac{7}{5}\log x + \frac{57}{5}\log\left(x-5\right)$$

$$\int_{dx} \frac{x^2+3x+1}{x^2-5} = x + \frac{3}{2}\log\left(x^2-5\right) + \frac{3}{\sqrt{5}}\log\left(\frac{\sqrt{5}-x}{\sqrt{5}+x}\right)$$

$$\int_{dx} \frac{7x+8}{x^2+3x} = \frac{8}{3}\log x + \frac{13}{3}\log\left(x+3\right)$$

$$\int_{dx} \frac{5x^4+8x+1}{x^2-7x} = \frac{5}{3}x^3 + \frac{35}{2}x^2 + 245x - \frac{1}{7}\log x + \frac{12062}{7}\log\left(x-7\right)$$

$$\int_{dx} \frac{2x-1}{\left(x^2-x+1\right)^3} = -\frac{1}{2}\left(x^2-x+1\right)^{-2}$$

$$\int_{dx} \frac{x^3+7x^2+2x+1}{x^3+x^2-x-1} = x + \frac{11}{4}\log\left(x-1\right) + \frac{5}{2}\left(x+1\right)^{-1} + \frac{13}{4}\log\left(x+1\right)$$

$$\int_{dx} \frac{x^6+7x^2+8}{x^5-4x^3+x^2-4} \frac{1}{2}x^2 - \frac{38\sqrt{3}}{63}\arctan\left((2x-1)/\sqrt{3}\right) - \frac{2}{7}\log\left(x^2-x+1\right) + \frac{25}{9}\log(x-2) + \frac{25}{7}\log(x+2) - \frac{16}{9}\log(x$$

$$\int_{dx} \frac{x^2}{x^3-3x-2} = \frac{1}{3}(x+1)^{-1} + \frac{4}{9}\log(x-2) + \frac{5}{9}\log(x+1)$$

$$\int_{dx} \frac{8x}{3x^4-10x^3+10x-3} = -\log(x-1) + \frac{3}{8}\log(3x-1) + \frac{3}{8}\log(x-3) + \frac{1}{4}\log(x+1)$$

$$\int_{dx} \frac{2x+7}{x^4-x^2} = 7x^{-1} - 2\log x + \frac{9}{2}\log(x-1) - \frac{5}{2}\log(x+1)$$

$$\int_{dx} \frac{x^4+8x+7}{x^3+5x^2+6x} = \frac{1}{2}x^2 - 5x + \frac{7}{6}\log x + \frac{64}{3}\log(x+3) - \frac{7}{2}\log(x+2)$$

$$\int_{dx} \frac{1}{\left(x^2-1\right)\left(x^2-4\right)} = \frac{1}{6}\log\left(\frac{x+1}{x-1}\right) + \frac{1}{12}\log\left(\frac{x-2}{x+2}\right)$$

$$\int_{dx} \frac{1}{(x+1)^3\left(x^2+1\right)} = -\frac{163}{3}(x-4)^{-3} - \frac{129}{2}(x-4)^{-2} - 34(x-4)^{-1} + 3\log(x-4)$$

$$\int_{dx} \frac{1}{x^4+x^2} = \frac{1}{6}x^{-1} - \frac{5}{36}\log x - \frac{7}{4}\log(x-2) + \frac{26}{9}\log(x-3)$$

rational integrals quadratic

$$\frac{x+b}{\left(x^2+2bx+c\right)^n}$$

$$\int \begin{cases} \dfrac{x+b}{x^2+2bx+c} \\ \dfrac{x+b}{\left(x^2+2bx+c\right)^n} \end{cases} = \begin{cases} \log\left(x^2+2bx+c\right) \\ \dfrac{-1}{(n-1)\left(x^2+2bx+c\right)^{n-1}} \end{cases}$$

$$\frac{1}{\left(x^2+2bx+c\right)^n}$$

$$\int \left(x^2+2bx+c\right)^{-n} = \int \left((x+b)^2 + \left(c-b^2\right)\right)^{-n} \underset{t=\frac{x+b}{\sqrt{c-b^2}}}{=} \left(c-b^2\right)^{1/2-n} \int_{dt} \left(t^2+1\right)^{-n}$$

$$\begin{cases} \int\limits_{dx} \left(x^2+bx+c\right)^{-1} = \dfrac{1}{\sqrt{c-b^2}} \int\limits_{dt} \dfrac{1}{t^2+1} = \dfrac{1}{\sqrt{c-b^2}} \arctan t = \dfrac{1}{\sqrt{c-b^2}} \arctan \dfrac{x+b}{\sqrt{c-b^2}} \\ \int\limits_{dx} \left(x^2+2bx+c\right)^{-n} = \left(c-b^2\right)^{1/2-n} \int\limits_{dt} \left(t^2+1\right)^{-n} \end{cases}$$

$$\int\limits_{dx} \left(x^2+1\right)^{-n} = \frac{\left(x^2+1\right)^{1-n} x}{2n-2} + \frac{2n-3}{2n-2} \int\limits_{dx} \left(x^2+1\right)^{1-n}$$

$$\int\limits_{dx} \frac{1}{1+x^2} = \arctan x$$

$$\int\limits_{dx} \frac{2}{x^2+4} = \arctan\left(x/2\right)$$

$$\int\limits_{dx} \frac{3}{x^2+9} = \arctan\left(x/3\right)$$

$$\int\limits_{dx} \frac{\sqrt{7}}{x^2+7} = \arctan\left(x/\sqrt{7}\right)$$

$$\int\limits_{dx} \frac{\sqrt{15}}{3x^2+5} = \arctan\left(\sqrt{3}x/\sqrt{5}\right)$$

$$\int\limits_{dx} \frac{x}{x^4+2} = \frac{\sqrt{2}}{4} \arctan\left(x^2/\sqrt{2}\right)$$

$$\int\limits_{dx} \frac{3x^2+x+1}{x^2-2x+3} = 3x + \frac{7}{2}\log\left(x^2-2x+3\right) - \frac{\sqrt{2}}{2}\arctan\left(\frac{x-1}{\sqrt{2}}\right)$$

$$\int\limits_{dx} \frac{1}{x^2-2x+2} = \arctan\left(x-1\right)$$

$$\int\limits_{dx} \frac{1}{x^2+x+1} = \frac{2}{\sqrt{3}} \arctan\left(\left(2x+1\right)/\sqrt{3}\right)$$

$$\int\limits_{dx} \frac{1}{x^4+x^2+1} = \frac{7}{\sqrt{3}} \arctan\left(\left(2x^2+1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{3x+4}{x^2-4x+5} = \frac{3}{2}\log\left(x^2-4x+5\right) + 10\arctan\left(x-2\right)$$

$$\int_{dx} \frac{18}{\left(x^2+3\right)^2} = \frac{3x}{x^2+3} + \sqrt{3}\arctan\left(x/\sqrt{3}\right)$$

$$\int_{dx} \frac{1}{\left(x^2+4x+13\right)^3} = \frac{1}{36}\frac{x+2}{\left(x^2+4x+13\right)^2} + \frac{1}{216}\frac{x+2}{x^2+4x+13} + \frac{1}{648}\arctan\left(\left(x+2\right)/3\right)$$

$$\int_{dx} \frac{1}{\left(x^2+13\right)^3} = \frac{1}{52}\frac{x}{\left(x^2+13\right)^2} + \frac{3}{1352}\frac{x}{x^2+13} + \frac{3\sqrt{13}}{17576}\arctan\left(x/\sqrt{13}\right)$$

$$\int_{dx} \frac{2x+7}{x^2-4x+13} = \log\left(x^2-4x+13\right) + \frac{11}{3}\arctan\left(\left(x-2\right)/3\right)$$

$$\int_{dx} \frac{1}{x^2-6x+9} = \frac{1}{2}\log\left(x^2-5x+36\right) + \frac{19}{\sqrt{119}}\arctan\left(\left(2x-5\right)/\sqrt{119}\right)$$

$$\int_{dx} \frac{2x+3}{\left(x^2+x+1\right)^2} = \frac{1}{3}\frac{4x-1}{x^2+x+1} + \frac{8\sqrt{3}}{9}\arctan\left(\left(2x+1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{2x+1}{\left(x^2-x+1\right)^3} = \frac{1}{6}\frac{4x-5}{\left(x^2-x+1\right)^2} + \frac{2}{3}\frac{2x-1}{x^2-x+1} + \frac{8\sqrt{3}}{9}\arctan\left(\left(2x-1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{5x+7}{x^2-5x+36} = \frac{5}{2}\log\left(x^2-5x+36\right) + \frac{39}{\sqrt{119}}\arctan\left(\left(2x-5\right)/\sqrt{119}\right)$$

$$\int_{dx} \frac{7x+5}{\left(x^2+x+1\right)^2} = \frac{x-3}{x^2+x+1} + \frac{2}{\sqrt{3}}\arctan\left(\sqrt{3}\left(2x+1\right)\right)$$

$$\int_{dx} \frac{8x^2+5x+7}{\left(x^2+4x+13\right)^2} = -\frac{1}{18}\frac{43x-157}{x^2+4x+13} + \frac{101}{54}\arctan\left(\left(x+2\right)/2\right)$$

$$\int_{dx} \frac{x^2+2x+7}{x^3-1} = \frac{10}{3}\log\left(x-1\right) - \frac{7}{6}\log\left(x^2+x+1\right) - \frac{5}{\sqrt{3}}\arctan\left(\left(2x+1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{3x^2+5x+1}{x^3+1} = -\frac{1}{3}\log\left(x+1\right) + \frac{5}{3}\log\left(x^2-x+1\right) - 2\sqrt{3}\arctan\left(\left(2x-1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{1}{\left(x^2+1\right)^2(x-1)} = \frac{1}{4}\log(x-1) - \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{2}\arctan x - \frac{1}{4}\frac{x-1}{x^2+1}$$

$$\int_{dx} \frac{x^4-x^2}{x^2+1} = \frac{1}{3}x^3 - 2x + 2\arctan x$$

$$\int_{dx} \frac{x^4+x^2+1}{\left(x^2+4\right)^2} = x + \frac{13}{8}\frac{x}{x^2+4} - \frac{43}{16}\arctan(x/2)$$

$$\int_{dx} \frac{5x^2+1}{\left(x^2+1\right)\left(x^2+5\right)} = -\arctan x + \frac{6}{\sqrt{5}}\arctan\left(x/\sqrt{5}\right)$$

$$\int_{dx} \frac{x^4+1}{x^4-1} = x + \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) - \arctan x$$

$$\int_{dx} \frac{x^4-1}{x^4+1} = x - \frac{\sqrt{2}}{4}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right) - \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}x+1\right) - \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}x-1\right)$$

$$\int_{dx} \frac{x^5+4x+1}{(x-1)^3\left(x^2+x+1\right)} = x - (x-1)^{-2} - (x-1)^{-1} + \frac{5}{3}\log(x-1) + \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{\sqrt{3}}\arctan\left((2x+1)/\sqrt{3}\right)$$

$$\int_{dx} \frac{x+1}{x^5-x^4+x^3} = -\frac{1}{2}x^{-2} - 2x^{-1} + \log x - \frac{1}{2}\log\left(x^2-x+1\right) - \sqrt{3}\arctan\left((2x-1)/\sqrt{3}\right)$$

$$\int_{dx} \frac{32}{x^4+16} = \frac{1}{\sqrt{2}}\log\left(\frac{x^2+2\sqrt{2}x+4}{x^2-2\sqrt{2}x+4}\right) + \sqrt{2}\arctan\left(1+x/\sqrt{2}\right) + \sqrt{2}\arctan\left(-1+x/\sqrt{2}\right)$$

$$\int_{dx} \frac{3x^3-2x^2+x-1}{(x-4)^4} = -x^{-1} - \arctan x$$

$$\int_{dx} \frac{x^3-1}{x^4-5x^3+6x^2} = \frac{13}{18}\log(x-1) - \frac{3}{2}\log(x+1) + \frac{7}{18}\log\left(x^2+x+1\right) + \frac{\sqrt{3}}{9}\arctan\left((2x+1)/\sqrt{3}\right) + \frac{2}{3}\frac{3x+2}{x^2+x+1}$$

$$\int_{dx} \frac{7x^2+5x+1}{\left(x^2-1\right)\left(x^2+x+1\right)^2} = -\frac{5}{6}\log\left(x^2+2\right) - \frac{5\sqrt{2}}{6}\arctan\left(x/\sqrt{2}\right) + \frac{5}{6}\log\left(x^2+x+1\right) + \frac{7}{\sqrt{3}}\arctan\left((2x+1)/\sqrt{3}\right)$$

$$\int_{dx} \frac{x^2+7}{\left(x^2+2\right)\left(x^2+x+1\right)} = \frac{1}{4}\log\left(x^2+x+1\right) + \frac{\sqrt{3}}{6}\arctan\left((2x+1)/\sqrt{3}\right) - \frac{1}{4}\log\left(x^2-x+1\right) + \frac{\sqrt{3}}{6}\arctan\left((2x-1)/\right.$$

$$\int_{dx} \frac{7x}{x^4+x^2+1} = -\arctan\left(2\left(x^2-4\right)^{-1/2}\right) + \log\left(x+\sqrt{x^2-4}\right)$$

Euler I integrals

$$\int_{dx} R\left(x{:}\left(\frac{ax+b}{cx+d}\right)^{1/n}\right) = n\left(ad-bc\right)\int_{dt} R\left(\frac{b-dt^n}{ct^n-a}{:}t\right)\frac{t^{n-1}}{\left(ct^n-a\right)^2} \text{ rat}$$

$$t = \left(\frac{ax+b}{cx+d}\right)^{1/n} \Rightarrow \begin{cases} x = \dfrac{b-dt^n}{ct^n-a} \\ dx = n\left(ad-bc\right)\dfrac{t^{n-1}}{\left(ct^n-a\right)^2}dt \end{cases}$$

$$\int_{dx} x\sqrt{\frac{x+2}{x-2}} =$$

$$\int_{dx} \frac{1}{x}\left(\frac{x-1}{x+1}\right)^{1/3}$$

$$\int_{dx} \left(\frac{2x-1}{3-5x}\right)^{1/3}$$

$$\int_{dx} x\sqrt{\frac{x+1}{x-2}} = \left(\frac{1}{2}x+\frac{7}{4}\right)\sqrt{x^2-x-2} + \frac{15}{8}\log\left(x+\sqrt{x^2-x-2}-1/2\right)$$

$$\int_{dx} \frac{1}{x}\sqrt{\frac{x+2}{x-2}} = \left(\frac{1}{2}x+2\right)\sqrt{x^2-4} + 2\log\left(x+\sqrt{x^2-4}\right)$$

Euler II integrals

$$\int\limits_{dx} R\left(x{:}\sqrt{x^2+2bx+c}\right) = \int\limits_{dt} R\left(\frac{t^2-c}{2\,(t+b)}{:}t-\frac{t^2-c}{2\,(t+b)}\right)\frac{t^2+2bt+c}{2{(t+b)}^2}\ \text{rat}$$

$$t = x+\sqrt{x^2+2bx+c} \Longrightarrow \begin{cases} x = \dfrac{t^2-c}{2\,(t+b)} \\ dx = dt\dfrac{t^2+2bt+c}{2{(t+b)}^2} \end{cases}$$

$$\int\limits_{dx} \frac{1}{\sqrt{x^2+k}} = \log\left(x+\sqrt{x^2+k}\right)$$

$$\int\limits_{dx} \sqrt{x^2+k}$$

$$\int\limits_{dx} \frac{x^2}{\sqrt{x^2+k}}$$

$$\int\limits_{dx} \frac{2x^2}{{\left(x^3-1\right)}^{1/3}} = \ {\left(x^3-1\right)}^{2/3}$$

$$\int\limits_{dx} \frac{x^{1/4}}{3+\sqrt{x}} = \frac{4}{3}\,x^{3/4}-12\,x^{1/4}+12\sqrt{3}\arctan\left(x^{1/4/\sqrt{3}}\right)$$

$$\int\limits_{dx} \frac{1}{x^{1/3}+\sqrt{x}} = 2\sqrt{x}-3\,x^{1/3}+6\,x^{1/6}-6\log\left(1+x^{1/6}\right)$$

$$\int\limits_{dx} \frac{x}{{(x-1)}^{1/3}} = \frac{3}{5}\,{(x-1)}^{5/3}+\frac{3}{2}\,{(x-1)}^{2/3}$$

$$\int\limits_{dx} \frac{3x}{x^{1/3}+\sqrt{x}} = 2\,x^{3/2}-\frac{9}{4}\,x^{4/3}+\frac{18}{7}\,x^{7/6}-3x+\frac{18}{5}\,x^{5/6}-\frac{9}{2}\,x^{2/3}+6\sqrt{x}-9\,x^{1/3}+18\,x^{1/6}-18\log\left(1+x^{1/6}\right)$$

$$\int\limits_{dx} \frac{x^{1/3}+1}{x^{1/3}-1} = x+3\,x^{2/3}+6\,x^{1/3}+6\log\left(x^{1/3}-1\right)$$

$$\int\limits_{dx} \frac{1}{\sqrt{x-1}+{(x-1)}^{1/4}} = 2\sqrt{x-1}-4\,{(x-1)}^{1/4}-4\log\left(1+{(x-1)}^{1/4}\right)$$

$$\int_{dx} \frac{2\sqrt{x}}{1-x^{4/3}} = -12\,x^{1/6} + \frac{3}{2}\log\left(\frac{x^{1/6}+1}{x^{1/6}-1}\right) + 3\arctan\left(x^{1/6}\right) + \frac{3\sqrt{2}}{4}\log\left(\frac{x^{1/3}+\sqrt{2}x^{1/6}+1}{x^{1/3}-\sqrt{2}x^{1/6}+1}\right) + \frac{3\sqrt{2}}{2}\arctan\left(\sqrt{2}x^{1/6}+1\right)$$

$$\int_{dx} \frac{1}{\sqrt{x^2+4x+4}} = \log\left(x+2\right)$$

$$\int_{dx} \frac{2x+7}{\sqrt{x^2+7x-1}} = 2\sqrt{x^2+7x-1}$$

$$\int_{dx} \frac{3x^2+1}{\sqrt{x^3+x+7}} = 2\sqrt{x^3+x+7}$$

$$\int_{dx} \frac{8x+2}{\sqrt{2x^2+x+1}} = 4\sqrt{2x^2+x+1}$$

$$\int_{dx} \frac{2-2x}{\sqrt{3+2x-x^2}} = 2\sqrt{3+2x-x^2}$$

$$\int_{dx} \frac{1}{\sqrt{x^2+3}} = \log\left(x+\sqrt{x^2+3}\right)$$

$$\int_{dx} \frac{\sqrt{2}}{\sqrt{2x^2+5}} = \log\left(x+\sqrt{x^2+5/2}\right)$$

$$\int_{dx} \frac{1}{\sqrt{x^2+x+1}} = \log\left(x+\sqrt{x^2+x+1}+1/2\right)$$

$$\int_{dx} \frac{3x+1}{\sqrt{x^2-x+1}} = 3\sqrt{x^2-x+1} + \frac{5}{2}\log\left(x+\sqrt{x^2-x+1}-1/2\right)$$

$$\int_{dx} \frac{5x+11}{\sqrt{4x^2+4x+3}} = \frac{5}{4}\sqrt{4x^2+4x+3} + \frac{17}{4}\log\left(x+\sqrt{x^2+x+3/4}+1/2\right)$$

$$\int_{dx} \frac{2}{\sqrt{4x^2+2x+1}} = \log\left(x+\sqrt{x^2+x/2+1/4}+1/4\right)$$

$$\int_{dx} \frac{1}{\sqrt{x^2+10x+11}} = \log\left(x+\sqrt{x^2+10x+11}+5\right)$$

$$\int_{dx} \frac{3x-7}{\sqrt{x^2+2x}} = 3\sqrt{x^2+2x} - 10\log\left(x+\sqrt{x^2+2x}+1\right)$$

$$\int_{dx} \frac{5x+2}{\sqrt{x^2+4x+5}} = 5\sqrt{x^2+4x+5} - 8\log\left(x+\sqrt{x^2+4x+5}+2\right)$$

$$\int_{dx} \frac{3x-2}{\sqrt{10x^2-7x}} = \frac{3}{10}\sqrt{10x^2-7x} - \frac{19\sqrt{10}}{200}\log\left(x/\sqrt{10}+\sqrt{10x^2-7x}-7/2\sqrt{10}\right)$$

$$\int_{dx} \frac{2x+7}{\sqrt{3x^2+5}} = \frac{2}{3}\sqrt{3x^2+5} + \frac{7}{\sqrt{3}}\log\left(x+\sqrt{x+5/3}\right)$$

$$\int_{dx} \frac{7x+1}{\sqrt{x^2+4x+7}} = 7\sqrt{x^2+4x+7} - 13\log\left(x+\sqrt{x^2+4x+7}+2\right)$$

$$\int_{dx} \frac{5x+7}{\sqrt{3x^2-2x}} = \frac{5}{3}\sqrt{3x^2-2x} + \frac{26\sqrt{3}}{9}\log\left(x/\sqrt{3}+\sqrt{3x^2-2x}-1/\sqrt{3}\right)$$

$$\int_{dx} \sqrt{x^2+9} = \frac{1}{2}x\sqrt{x^2+9} + \frac{9}{2}\log\left(x+\sqrt{x^2+9}\right)$$

$$\int_{dx} \sqrt{x^2-7x+12} = \left(\frac{1}{2}x-\frac{7}{4}\right)\sqrt{x^2-7x+12} - \frac{1}{8}\log\left(x+\sqrt{x^2-7x+12}-7/2\right)$$

$$\int_{dx} \frac{3x+5}{\sqrt{5-4x-x^2}} = -3\sqrt{5-4x-x^2} - \arcsin\left(\left(x+2\right)/3\right)$$

$$\int_{dx} \sqrt{x^2+12x-64} = \left(\frac{1}{2}x+3\right)\sqrt{x^2+12x-64} - 50\log\left(x+\sqrt{x^2+12x-64}+6\right)$$

$$\int_{dx} \sqrt{x^2-x-1} = \left(\frac{1}{2}x-\frac{1}{4}\right)\sqrt{x^2-x-1} - \frac{5}{8}\log\left(x+\sqrt{x^2-x-1}-1/2\right)$$

$$\int_{dx} x\sqrt{x^2-1} = \frac{1}{3}\left(x^2-1\right)^{3/2}$$

$$\int_{dx} x\sqrt{x^2+4} = \frac{1}{3}\left(x^2+4\right)^{3/2}$$

$$\int_{dx} x^2\sqrt{x^2+9} = \frac{1}{4}x\left(x^2+9\right)^{3/2} - \frac{9}{8}x\sqrt{x^2+9} - \frac{81}{8}\log\left(x+\sqrt{x^2+9}\right)$$

$$\int_{dx} \sqrt{4x^2+2x+1} = \left(\frac{1}{2}x+\frac{1}{8}\right)\sqrt{4x^2+2x+1}$$

$$\int_{dx} \frac{5x-11}{\sqrt{20+4x-x^2}} = -5\sqrt{20+4x-x^2} - \arcsin\left(\sqrt{6}\left(x-2\right)/12\right)$$

$$\int_{dx} \sqrt{8x^2+3x+2} = \left(\frac{1}{2}x+\frac{3}{32}\right)\sqrt{8x^2+3x+2} + \frac{55}{256}\log\left(x+\sqrt{x^2+3x/8+1/4}+3/16\right)$$

$$\int_{dx} \frac{x^2}{\sqrt{x^2+5}} = \frac{1}{2}x\sqrt{x^2+5} - \frac{5}{2}\log\left(x+\sqrt{x^2+5}\right)$$

$$\int_{dx} \frac{3x^2}{\sqrt{2x^2+7}} = \frac{3}{4}x\sqrt{2x^2+7} - \frac{21\sqrt{2}}{8}\log\left(x+\sqrt{x^2+7/2}\right)$$

$$\int_{dx} \frac{x^2+x}{\sqrt{2x^2+3}} = \left(\frac{1}{4}x+\frac{1}{2}\right)\sqrt{2x^2+3} - \frac{3\sqrt{2}}{8}\log\left(x+\sqrt{x^2+3/2}\right)$$

$$\int_{dx} \frac{x^2-x+1}{\sqrt{x^2+x+1}} = \left(\frac{1}{2}x-\frac{7}{4}\right)\sqrt{x^2+x+1} + \frac{11}{8}\log\left(x+\sqrt{x^2+x+1}+1/2\right)$$

$$\int_{dx} \frac{3x^2-2x-1}{\sqrt{x^2+4x-5}} = \left(\frac{3}{2}x-11\right)\sqrt{x^2+4x-5} + \frac{57}{2}\log\left(x+\sqrt{x^2+4x-5}+2\right)$$

$$\int_{dx} \frac{x^3+2x^2+5x+1}{\sqrt{x^2+2}} = \left(\frac{1}{3}x^2+x+\frac{11}{3}\right)\sqrt{x^2+2} - \log\left(x+\sqrt{x^2+2}\right)$$

$$\int_{dx} \frac{2x^3+x^2+1}{\sqrt{2x^2+5x-12}} = \left(\frac{1}{3}x^2-\frac{11}{24}x+\frac{223}{32}\right)\sqrt{2x^2+5x-12} - \frac{1659\sqrt{2}}{128}\log\left(\sqrt{2}x+\sqrt{2x^2+5x-12}+5/2\sqrt{2}\right)$$

$$\int_{dx} \frac{x^4}{\sqrt{x^2-x-1}} = \left(\frac{1}{4}x^3+\frac{7}{24}x^2+\frac{71}{96}x+\frac{325}{192}\right)\sqrt{x^2-x-1} + \frac{203}{128}\log\left(x+\sqrt{x^2-x-1}-1/2\right)$$

$$\int_{dx} \frac{2\sqrt{5}}{x\sqrt{x^2+5}} = -\log\left(\frac{\sqrt{x^2+5}+\sqrt{5}}{\sqrt{x^2+5}-\sqrt{5}}\right)$$

$$\int_{dx} \frac{1}{x^2\sqrt{x^2+x+1}} = -\frac{x^2+x+1}{x} + \frac{1}{4}\log\left(\frac{\sqrt{x^2+x+1}+1+x/2}{\sqrt{x^2+x+1}-1-x/2}\right)$$

$$\int_{dx} \frac{1}{(x+1)\sqrt{x^2+x}} = 2\sqrt{\frac{x}{x+1}}$$

$$\int_{dx} \frac{4\sqrt{2}}{(x-1)\sqrt{x^2+7}} = -\log\left(\frac{2\sqrt{2}\sqrt{x^2+7}+x+7}{2\sqrt{2}\sqrt{x^2+7}-x-7}\right)$$

$$\int_{dx} \frac{1}{x^2\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}}{x}$$

$$\int_{dx} \frac{1}{(x-1)\sqrt{x^2+x+1}} =$$

Euler III integrals

$$\int_{dx} \sqrt{a^2-x^2}$$

$$\int_{dx} \frac{x^2}{\sqrt{a^2-x^2}}$$

$$\int_{dx} \frac{\overbrace{1-x^2}^{-1/2} \;\Big|\; \overbrace{x^2+1}^{-1/2}}{\overbrace{1-x^2}^{-1/2} \;\Big|\; \overbrace{x^2-1}^{-1/2}} = \frac{\arcsin x \;\Big|\; \overset{-1}{\sinh} x}{-\overset{-1}{\cos} x \;\Big|\; \overset{-1}{\cosh} x}$$

$$\int_{dx} \frac{x}{\sqrt{1-x^2}} = -\sqrt{1-x^2}$$

$$\int_{dx} \frac{2x}{\sqrt{1-x^4}} = \arcsin\left(x^2\right)$$

$$\int_{dx} \frac{2}{\sqrt{1-4x^2}} = \arcsin(2x)$$

$$\int_{dx} \frac{9x-13}{\sqrt{-x^2-5x-6}} = -9\sqrt{-x^2-5x-6} - \frac{71}{2}\arcsin(2x+5)$$

$$\int_{dx} \sqrt{8-x^2} = \frac{1}{2}x\sqrt{8-x^2} + 4\arcsin\left(\sqrt{2}x/4\right)$$

$$\int_{dx} \sqrt{2+5x-3x^2} = \left(\frac{1}{2}x - \frac{5}{12}\right)\sqrt{2+5x-3x^2} + \frac{49\sqrt{3}}{72}\arcsin\left(\left(6x-5\right)/7\right)$$

$$\int_{dx} \frac{\sqrt{3}}{\sqrt{5-3x^2}} = \arcsin\left(\sqrt{3}x/\sqrt{5}\right)$$

$$\int_{dx} \sqrt{45+4x-x^2} = \left(\frac{1}{2}x - 1\right)\sqrt{45+4x-x^2} + \frac{49}{2}\arcsin\left(\left(x-2\right)/7\right)$$

$$\int_{dx} \frac{1}{\sqrt{5x-x^2-6}} = \arcsin\left(2x-5\right)$$

$$\int_{dx} \frac{1}{\sqrt{2-2x-x^2}} = \arcsin\left(\left(x+1\right)/\sqrt{3}\right)$$

$$\int_{dx} \frac{2x+7}{\sqrt{4-x^2}} = -2\sqrt{4-x^2} + 7\arcsin\left(x/2\right)$$

$$\int_{dx} \frac{7x+10}{\sqrt{4x-x^2}} = -7\sqrt{4x-x^2} + 24\arcsin\left(\left(x-2\right)/2\right)$$

$$\int_{dx} \frac{3x^2}{\sqrt{5-4x^2}} = -\frac{3}{8}x\sqrt{5-4x^2} + \frac{15}{16}\arcsin\left(2x/\sqrt{5}\right)$$

$$\int_{dx} \frac{7x^2+1}{\sqrt{9-x^2}} = -\frac{7}{2}x\sqrt{9-x^2} + \frac{65}{2}\arcsin\left(x/3\right)$$

$$\int_{dx} \frac{3-2x^2}{\sqrt{1-4x^2}} = \frac{11}{8}\arcsin\left(2x\right) + \frac{1}{4}x\sqrt{1-4x^2}$$

$$\int_{dx} \frac{x^2+2x-1}{\sqrt{5-2x-x^2}} = -\left(\frac{1}{2}x + \frac{1}{2}\right)\sqrt{5-2x-x^2} + \arcsin\left(\left(x+1\right)/\sqrt{6}\right)$$

$$\int_{dx} \frac{2x^2+x+5}{\sqrt{3+2x-x^2}} = -\left(x+4\right)\sqrt{3+2x-x^2} + 12\arcsin\left(\left(x-1\right)/2\right)$$

$$\int\limits_{dx} \frac{x^3+2x+1}{\sqrt{4x-x^2}} = -\left(\frac{1}{3}x^2+\frac{5}{3}x+12\right)\sqrt{4x-x^2}+25\arcsin\left(\left(x-2\right)/2\right)$$

$$\int\limits_{dx} \frac{1}{x^2\sqrt{9-x^2}} = -\frac{\sqrt{9-x^2}}{9x}$$

$$\int\limits_{dx} \frac{1}{x^3\sqrt{3+2x-x^2}} = \frac{x-1}{6x^2}\sqrt{3+2x-x^2}-\frac{\sqrt{3}}{18}\log\left(\frac{\sqrt{3+2x-x^2}+\sqrt{3}+x/\sqrt{3}}{\sqrt{3+2x-x^2}-\sqrt{3}-x/\sqrt{3}}\right)$$

$$\int\limits_{dx} \frac{1}{x^2\sqrt{1-x^2}} = -\frac{\sqrt{1-x^2}}{x}$$

log integrals

$$\int\limits_{dy} \log^n y = \log^n y y - n\int\limits_{dy} \log^{n-1} y$$

$$\int\limits_{dy} \log^n y\, y^m = \frac{\log^n y y^{m+1}}{m+1}-\frac{n}{m+1}\int\limits_{dy} \log^{n-1} y\, y^m$$

$$\int\limits_{dy} \frac{\log^n y}{y} \underset{x=\log y}{=} \int\limits_{dx} x^n$$

$$\int\limits_{dy} y\log y = \frac{1}{2}y^2\log y-\frac{1}{4}y^2$$

$$\int\limits_{dy} y^3\log\left(2y\right) = \frac{1}{4}y^4\log\left(2y\right)-\frac{1}{16}y^4$$

$$\int\limits_{dy} \frac{\log y}{\sqrt{y}} = 2\sqrt{y}\left(\log y-2\right)$$

$$\int\limits_{dy} \sqrt{y}\log y = \frac{2}{3}\left(\log y-\frac{2}{3}\right)y^{3/2}$$

$$\int\limits_{dy} \log^2 y$$

$$\int\limits_{dy} \log(4y-1) = \left(y - \frac{1}{4}\right) \log(4y-1) - y$$

$$\int\limits_{dy} y \log(2y+5) = \left(\frac{1}{2}y^2 - \frac{25}{8}\right) \log(2y+5) - \frac{1}{4}y^2 + \frac{5}{4}y$$

$$\int\limits_{dy} y^2 \log y = \frac{1}{3}y^3 \log y - \frac{1}{9}y^3$$

$$\int\limits_{dy} y \log^2 y$$

$$\int\limits_{dy} y^2 \log(y+2) = \frac{1}{3}\left(y^3+1\right) \log(y+1) - \frac{1}{18}y\left(2y^2-3y+6\right)$$

$$\int\limits_{dy} \frac{\log(\log y)}{y} = (\log(\log y) - 1) \log y$$

$$\int\limits_{dy} \log(3y+7) = \left(y + \frac{7}{3}\right) \log(3y+7) - y$$

$$\int\limits_{dy} y^3 \log\left(3y^2+1\right) = \frac{1}{4}\left(y^4 - \frac{1}{9}\right) \log\left(3y^2+1\right) - \frac{1}{8}y^4 + \frac{1}{12}y^2$$

$$6\int\limits_{dy} \frac{\log^5 y}{y} = \log^6 y$$

$$\int\limits_{dy} \frac{1}{y \log y} = \log(\log y)$$

$$\int\limits_{dy} \frac{1}{y \log y \log(\log y)} = \log(\log(\log y))$$

$$\int\limits_{dy} \frac{\log^2 y}{y^2} = -\frac{\log^2 y}{y} - 2\frac{\log y}{y} - 2y^{-1}$$

$$\int_{dy} \log\left(y^2+1\right) = y\log\left(y^2+1\right) - 2y + 2\arctan y$$

$$\int_{dy} \log^3 y = y\log^3 y - 3y\log^2 y + 6y\log y - 6y$$

$$\int_{dy} y^2\,\log(y+1)$$

$$\int_{dy} \frac{\log^3 y}{y^2}$$

$$\int_{dy} \log\left(y+\sqrt{y^2+1}\right) = y\log\left(y+\sqrt{y^2+1}\right) - \sqrt{y^2+1}$$

$$\int_{dy} y\log\left(y+\sqrt{y^2+1}\right) = \left(y^2+\frac{1}{2}\right)\log\left(y+\sqrt{y^2+1}\right) - \frac{1}{2}y\sqrt{y^2+1}$$

$$\int_{dx} \sin(\log x)$$

$$\int_{dx} \cos x\log(\cot x)$$

improper integrals

$$\begin{cases} \int_{dx}^{1|\infty} \frac{1}{x^{4/3}} \\ \int_{dx}^{0|1} \frac{1}{\sqrt{x}} = 2 \\ \int_{dx}^{-1|1} \frac{1}{x^{5/3}} \\ \int_{dx}^{1|\infty} x^{-5/2} \end{cases}$$

$$\int_{dx}^{1|\infty} \frac{3x}{\left(1+x^2\right)^2} \colon \int_{dx}^{-\infty|0} \frac{1}{x^2+4} \colon \int_{dx}^{0|5} \frac{1}{x^2-4x-5} = \infty \colon \int_{dx}^{2|3} \frac{1}{x^2-2x-3} \colon \int_{dx}^{-1|\infty} \frac{1}{x^2+4x+5}$$

$$\int_{dx}^{0|6} \frac{2x}{x^2-4} \text{ never integrate across singularity}$$

$$\int_{dx}^{1|\infty} (x+2)^{-1/3} \underset{u=x+2}{=} \int_{du}^{3|\infty} u^{-1/3} = \begin{cases} \frac{3}{2}u^{2/3} \\ 3|\infty \end{cases} \quad \text{div}$$

$$\begin{cases} \int_{dx}^{-3|3} \frac{1}{\sqrt{9-x^2}} \\ \int_{dx}^{0|3} \frac{1}{\sqrt{9-x^2}} \end{cases}$$

$$\int_{dx}^{1|\infty} x \underbrace{e^{-x}}_{=g'} = x\frac{e^{-x}}{-1} - \int_{dx}^{1|\infty} \frac{e^{-x}}{-1} = \begin{cases} -xe^{-x} - e^{-x} \\ 1|\infty \end{cases} \quad = -\left(-\frac{1}{e} - \frac{1}{e}\right) = \frac{2}{e}$$

$$\int_{dx}^{0|\infty} e^{-x} \colon \int_{dx}^{1|\infty} x \exp(-2x)$$

$$\begin{cases} \int_{dx}^{1|\infty} \frac{1}{x\sqrt{\log x}} \\ \int_{dx}^{1|\infty} \frac{1}{x \log x} = +\infty \\ \int_{dx}^{1|\infty} \frac{\log x}{x} \\ \int_{dx}^{2|\infty} \frac{1}{x \log x^2} = \frac{1}{\log 2} \end{cases}$$

$$\int_{dx}^{0|\infty} \arctan x = \infty \text{ increasing}$$

Beta integral

$$p \geqslant 0 \leqslant q: \quad \int\limits_{dx}^{0|1} x^p \, \overparen{1-x}^{q} = \frac{p!q!}{(p+q+1)!} =: B\,(p+1:q+1)$$

$$0 = p: \quad \int\limits_{dx}^{0|1} \overparen{1-x}^{q} = -\begin{cases} \dfrac{\overparen{1-x}^{q+1}}{q+1} \\ 0|1 \end{cases} = \frac{1}{q+1} = \frac{q!}{(q+1)!}$$

$$0 \leqslant p \curvearrowright p+1: \quad \int\limits_{dx}^{0|1} x^{p+1} \, \overparen{1-x}^{q} = -\int\limits_{dx}^{0|1} x^{p+1} \frac{d}{dx} \frac{\overparen{1-x}^{q+1}}{q+1} = \int\limits_{dx}^{0|1} \frac{d}{dx} x^{p+1} \frac{\overparen{1-x}^{q+1}}{q+1} - \underbrace{\begin{cases} x^{p+1} \dfrac{\overparen{1-x}^{q+1}}{q+1} \\ 0|1 \end{cases}}_{=0}$$

$$= \frac{p+1}{q+1} \int\limits_{dx}^{0|1} x^p \, \overparen{1-x}^{q+1} \underset{\text{ind}}{=} \frac{p+1}{q+1} \frac{p!\,(q+1)!}{(p+q+2)!} = \frac{(p+1)!q!}{(p+q+2)!}$$

trig substitution

$$\int\limits_{dt} \frac{\sin t \mid \sinh t}{\cos t \mid \cosh t} = \frac{-\cos t \mid \cosh t}{\sin t \mid \sinh t}$$

$$\int \sin t = -\cos t$$

$$\int \cos t = \sin t$$

$$\text{Sub} \quad \int\limits_{dt}^{0|\sqrt{l}p} \sin\left(t^2 + \pi\right) t \underset{u = t^2 + \pi}{=} \frac{1}{2} \int\limits_{du}^{\pi|2\pi} \sin u = -\frac{1}{2} \cos(\pi|2\pi) = -1$$

$$\text{Sub} \quad \int\limits_{dt} t \cos\left(t^2\right) \underset{u = t^2}{=} \int\limits_{du} \cos u \models \sin u/2 = \sin\left(t^2\right)/2$$

$$8 \int\limits_{dt} \sin(8t) = -\cos(8t)$$

$$7\int\limits_{dt} \cos(7t) = \sin(7t)$$

$$3\int\limits_{dt} \sin t \cos^2 t = -\cos^3 t$$

$$2\int\limits_{dx} \sin(2x+5) = -\cos(2x+5)$$

$$6\int\limits_{dx} x\sin\left(3x^2-1\right) = -\cos\left(3x^2-1\right)$$

$$6\int\limits_{dx} x\cos\left(3x^2-1\right) = \sin\left(3x^2-1\right)$$

$$7\int\limits_{dx} \sin^6 x \cos x = \sin^7 x$$

$$8\int\limits_{dx} \frac{\tan^7 x}{\cos^2 x} = \tan^8 x$$

$$\int\limits_{dx} \frac{\sin x}{1+\cos x} = -\log(1+\cos x)$$

$$\int\limits_{dt} \frac{\sin t}{\cos^{4/3} t} = 3\cos^{-1/3} t$$

$$\int\limits_{dt} \frac{\cos t}{\sqrt{1+\sin^2 t}} = \log\left(\sin t + \sqrt{1+\sin^2 t}\right)$$

$$2\int\limits_{dx} \sin x \cos x = \sin^2 x$$

trig partielle Integration

$$\int\limits_{dx} x\cos x = \cos x + x\sin x$$

$$\int\limits_{dx} x \sin(3x) = \frac{1}{9}\sin(3x) - \frac{1}{3}x\cos(3x)$$

$$\int\limits_{dx} (x+1)\sin(x+2) = \sin(x+2) - (x+1)\cos(x+2)$$

$$\int\limits_{dx} x^2 \sin x = \left(2 - x^2\right)\cos x + 2x\sin x$$

$$\int\limits_{dx} x^3 \cos(2x) = \left(\frac{1}{2}x^3 - \frac{3}{4}x\right)\sin(2x) + \left(\frac{3}{4}x^2 - \frac{3}{8}\right)\cos(2x)$$

$$\int\limits_{dx} x^5 \cos(4x) = \left(\frac{1}{4}x^5 - \frac{5}{16}x^3 + \frac{15}{128}x\right)\sin(4x) + \left(\frac{5}{16}x^4 - \frac{15}{64}x^2 + \frac{15}{512}\right)\cos(4x)$$

$$\int\limits_{dx} x^2 \exp(3x) = \left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right)\exp(3x)$$

$$\int\limits_{dx} \left(2 - x^2\right)\exp(3x) = \left(\frac{16}{27} + \frac{2}{9}x - \frac{1}{3}x^2\right)\exp(3x)$$

$$\int\limits_{dx} x^5 \exp x = \left(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120\right)\exp x$$

$$\int\limits_{dx} (x+2)\ 3^x = \frac{(x+2)\,3^x}{\log 3} - 3^x \log^2 3$$

$$\int\limits_{dx} x\,2^x = -\frac{2^x}{\log^2 2} + \frac{x2^x}{\log 2}$$

$$\int\limits_{dx} \exp x \sin x = \frac{1}{2}\exp x\,(\sin x - \cos x)$$

$$\int\limits_{dx} \exp x \cos x = \frac{1}{2}\exp x\,(\sin x + \cos x)$$

$$13\int\limits_{dx} \exp(2x)\sin(3x) = \exp(2x)\,(2\sin(3x) - 3\cos(3x))$$

$$10 \int\limits_{dx} \exp(3x)\cos x = \exp(3x)(\sin x + 3\cos x)$$

$$34 \int\limits_{dx} \exp x \sin(5x) = \exp(3x)(3\sin(5x) - 5\cos(5x))$$

$$2 \int\limits_{dx} x \arctan x = x^2 \arctan x - x + \arctan x$$

$$\int\limits_{dx} (x+5)\arctan x = \left(\frac{1}{2}x^2 + 5x + \frac{1}{2}\right)\arctan x - \frac{1}{2}x - \frac{5}{2}\log\left(x^2+1\right)$$

$$\int\limits_{dx} x^2 \arctan(2x) = \frac{1}{3}x^3 \arctan(2x) - \frac{1}{12}x^2 + \frac{1}{48}\log\left(1+4x^2\right)$$

$$\int\limits_{dx} \frac{x}{\cos^2 x} = x\tan x + \log(\cos x)$$

$$\text{part} \int\limits_{dt} \begin{cases} t^2 \sin t \\ t\cos t \end{cases}$$

trig powers

$$\int_{dt} \tan^n t = \frac{\tan^{n-1}}{n-1} - \int_{dt} \tan^{n-2} t \begin{cases} \int_{dt} \tan^{2m} t = \dfrac{\tan^{2m-1}}{2m-1} - \int_{dt} \tan^{2m-2} t \\ \int_{dt} \tan^{2m+1} t = \dfrac{\tan^{2m}}{2m} - \int_{dt} \tan^{2m-1} t \end{cases}$$

$$\int_{dt} \tan^2 (2t) = \frac{1}{2} \tan(2t) - t$$

$$\int_{dt} \tan^3 t = \frac{1}{2} \tan^2 t - \frac{1}{2} \log\left(1 + \tan^2 t\right)$$

$$\int_{dt} \tan^4 (5t) = \frac{1}{15} \tan^3 (5t) - \frac{1}{5} \tan(5t) + t$$

$$\int_{dt} \cot^2 t = -\cot t - t$$

$$\int_{dt} \cot^3 t = -\frac{1}{2} \cot^2 t + \frac{1}{2} \log\left(1 + \cot^2 t\right)$$

$$\int_{dt} \cot^4 t = \cot t - \frac{1}{3} \cot^3 t + t$$

$$\int\limits_{dt} \sin^n t = -\frac{\sin^{n-1}\cos}{n} + \frac{n-1}{n}\int\limits_{dt} \sin^{n-2} t \begin{cases} \int\limits_{dt} \sin^{2m} t = -\frac{\sin^{2m-1}\cos}{2m} + \frac{m-1/2}{m}\int\limits_{dt} \sin^{2m-2} t \\ \int\limits_{dt} \sin^{2m+1} t = -\frac{\sin^{2m}\cos}{2m+1} + \frac{m}{m+1/2}\int\limits_{dt} \sin^{2m-1} t \end{cases}$$

$$\int\limits_{dt} \sin^2(3t) = -\frac{1}{6}\cos(3t)\sin(3t) + \frac{1}{2}t$$

$$\int\limits_{dt} \sin^3 t = \frac{1}{3}\left(2 - \sin^2 t\right)\cos t$$

$$\int\limits_{dt} \sin^4 t = -\frac{1}{8}\left(3 + 2\sin^2 t\right)\sin t\cos t + \frac{3}{8}t$$

$$\int\limits_{dt} \sin^5(3t) = \frac{1}{45}\left(8 + 4\sin^2(3t) - 3\sin^4(3t)\right)\cos(3t)$$

$$\int\limits_{dt} \sin^6(3t) = -\left(\frac{5}{48} + \frac{5}{72}\sin^2(3t) + \frac{1}{18}\sin^4(3t)\right)\sin(3t)\cos(3t) + \frac{5}{16}t$$

$$\int\limits_{dt} \frac{3}{\sin(3t)} = \log(\tan(3t/2))$$

$$\int\limits_{dt} \frac{30}{\sin(30t)} = \log(\tan(15t))$$

$$\int\limits_{dt} \frac{2}{\sin^2(2t)} = -\cot(2t)$$

$$\int\limits_{dt} \frac{1}{\sin^3(4t)}$$

$$\int\limits_{dt} \frac{4}{\sin^3(2t)} = -\frac{\cos(2t)}{\sin^2(2t)} + \log(\tan t)$$

$$\int\limits_{dt} \frac{1}{\sin^5(2t)} = -\frac{1}{8}\frac{\cos(2t)}{\sin^4(2t)} - \frac{3}{16}\frac{\cos(2t)}{\sin^2(2t)} + \frac{3}{16}\log(\tan t)$$

$$\int {}^{t}\sin^{-2} = -\cot t$$

$$\int {}^{t}\sin^{-1} = \int \frac{1}{\sin(t/2)\cos(t/2)}$$

$$\int\limits_{dt} \cos^n t = \frac{\cos^{n-1}\sin}{n} + \frac{n-1}{n}\int\limits_{dt} \cos^{n-2} t \begin{cases} \int\limits_{dt} \cos^{2m} t = \frac{\cos^{2m-1}\sin}{2m} + \frac{m-1/2}{m}\int\limits_{dt} \cos^{2m-2} t \\ \int\limits_{dt} \cos^{2m+1} t = \frac{\cos^{2m}\sin}{2m+1} + \frac{m}{m+1/2}\int\limits_{dt} {}^{t}\cos^{2m-1} \end{cases}$$

$$\int\limits_{dt} \cos^2(4t) = \frac{1}{8}\cos(4t)\sin(4t) + \frac{1}{2}t$$

$$\int\limits_{dt} \cos^3(3t) = \frac{1}{9}\left(2 + \cos^2(3t)\right)\sin(3t)$$

$$\int\limits_{dt} \cos^4(2t) = \frac{1}{16}\left(3 + 2\cos^2(2t)\right)\sin(2t)\cos(2t) + \frac{3}{8}t$$

$$\int\limits_{dt} \cos^5 t = \frac{1}{15}\left(8 + 4\cos^2 t + 3\cos^4 t\right)\sin t$$

$$\int\limits_{dt} \cos^6 t = \left(\frac{5}{16} + \frac{5}{24}\cos^2 t + \frac{1}{6}\cos^4 t\right)\sin t\cos t + \frac{5}{16}t$$

$$\int\limits_{dt} \frac{2}{\cos(2t)} = \log(\tan(t + \pi/4))$$

$$\int\limits_{dt} \frac{7}{\cos(7t)} = \log(\tan(7t/2 + \pi/4))$$

$$\int\limits_{dt} \frac{3}{\cos^2(3t)} = \tan(3t)$$

$$\int\limits_{dt} \frac{2}{\cos^3 t} = \frac{\sin t}{\cos^2 t} + \log(\tan(t/2 + \pi/4))$$

$$\int\limits_{dt} \frac{1}{\cos^4 t} = \frac{1}{3}\frac{\sin t}{\cos^3 t} + \frac{2}{3}\frac{\sin t}{\cos t}$$

$$\int {}^{t}\cos^{-2} = \tan t$$

$$\int\limits_{dt} \sin^4 t + \cos^4 t = \frac{1}{4}\left(\cos^2 t - \sin^2 t\right)\sin t\cos t + \frac{3}{4}$$

$$\int_{dt} \sin^6 t + \cos^6 t = \frac{5}{24}\left(\cos^2 t - \sin^2 t\right)\sin t\cos t + \frac{1}{6}\left(\cos^4 t - \sin^4 t\right)\sin t\cos t + \frac{5}{8}t$$

$$\int_{dx} \cos^2(2x) = \frac{1}{4}\cos(2x)\sin(2x) + \frac{1}{2}x$$

$$\int_{dx} \sin^2(5x) = -\frac{1}{10}\cos(5x)\sin(5x) + \frac{1}{2}x$$

$$6\int_{dx} \sin^3(2x) = -\cos(2x)\left(2 + \sin^2(2x)\right)$$

trig products

$$\frac{\sin(2x) + \sin(2y)}{2} = \sin(x+y) + \cos(x-y)$$

$$\int_{dx} \sin x\cos(2x) = -\frac{1}{6}\cos(3x) + \frac{1}{2}\cos x$$

$$\int_{dx} \sin(4x)\cos(5x) = -\frac{1}{18}\cos(9x) + \frac{1}{2}\cos x$$

$$\int_{dx} \sin(3x)\sin(5x) = \frac{1}{4}\sin(2x) - \frac{1}{16}\sin(8x)$$

$$\int_{dx} \sin(4x)\sin(7x) = \frac{1}{6}\sin(3x) - \frac{1}{22}\sin(11x)$$

$$\int_{dx} \cos(4x)\cos(7x) = \frac{1}{6}\sin(3x) + \frac{1}{22}\sin(11x)$$

$$\int_{dx} \cos(11x)\cos(12x) = \frac{1}{2}\sin x + \frac{1}{46}\sin(23x)$$

trig integrals

$$\int_{dt}^{-\pi|\pi} R(\cos t : \sin t) = \int_{du}^{\mathbb{R}} R\left(\frac{2u}{1+u^2} : \frac{1-u^2}{1+u^2}\right) \frac{2}{1+u^2}$$

$$t \in -\pi|\pi$$

$$u = \tan(t/2) \Longrightarrow \begin{cases} dt \\ \cos t \\ \sin t \end{cases} = \frac{1}{1+u^2} \begin{cases} 2du \\ 2u \\ 1-u^2 \end{cases}$$

$$\int_{dt} R\left(\cos^2 t : \cos t \sin t : \sin^2 t\right) = \int_{dv} R\left(\frac{1}{1+v^2} : \frac{v}{1+v^2} : \frac{v^2}{1+v^2}\right) \frac{1}{1+v^2}$$

$$v = \tan t \Longrightarrow \begin{cases} dt \\ \cos^2 t \\ \cos t \sin t \\ \sin^2 t \end{cases} = \frac{1}{1+v^2} \begin{cases} dv \\ 1 \\ v \\ v^2 \end{cases}$$

$$8 \int_{dt} \sin^2 t \cos^2 t = t + \left(1 - 2\cos^2 t\right) \sin t \cos t$$

$$30 \int_{dt} \sin^2(2t) \cos^3(2t) = \left(2 + \cos^2(2t) - 3\cos^4(2t)\right) \sin(2t)$$

$$63 \int_{dt} \sin^3 t \cos^6 t = -\left(2 + 7\sin^2 t\right) \cos^7 t$$

$$12 \int_{dt} \sin^3 t \cos^3 t = -\left(1 + 2\sin^2 t\right) \cos^4 t$$

$$30 \int_{dt} \sin^3(2t) \cos^2(2t) = -\left(2 + 3\sin^2(2t)\right) \cos^3(2t)$$

$$\int_{dt} \sin(4t) \cos^2(2t) = -\frac{1}{32} \cos(8t) - \frac{1}{8} \cos(4t)$$

$$\int_{dt} \sin^2 t \cos^2(4t) = -\frac{1}{80}\sin(10t) + \frac{1}{32}\sin(8t) - \frac{1}{48}\sin(6t) - \frac{1}{8}\sin(2t) + \frac{1}{4}t$$

$$\int_{dt} \sin t \tan^2 t = \frac{\sin^4 t}{\cos t} + \sin^2 t \cos t + 2\cos t$$

$$2\int_{dt} \cos t \cot^3 t = -\frac{\cos^5 t}{\sin^2 t} - \cos^3 t - 3\cos t + 3\log(\sin t) - 3\log(\cos t - 1)$$

$$\int_{dt} \frac{\sin(2t)}{\sqrt{1-\cos^2 t}} = 2\sqrt{1-\cos^2 t}$$

$$\int_{dt} \frac{\sin(2t)}{\left(1+\sin^2 t\right)^{1/3}} = \frac{3}{2}\left(2-\cos^2 t\right)^{2/3}$$

$$\int_{dt} \frac{1}{\sin t \cos t} = \log(\tan t)$$

$$\int_{dt} \frac{2}{\sin^2 t \cos^3 t} = \frac{1}{\sin t \cos^2 t} - 3\frac{1}{\sin t} + 3\log(\tan(t/2 + \pi/4))$$

$$\int_{dt} \frac{1}{\sin^2 t \cos^2 t} = \frac{1}{\sin t \cos t} - 2\frac{\cos t}{\sin t}$$

$$5\int_{dt} \sin t \cos^4 t = -\cos^2 t$$

$$\int_{dt} \frac{2}{\sin^2 t \cos^3 t} = \frac{1}{\sin t \cos^2 t} - 3\frac{1}{\sin t} + 3\log(\tan t)$$

$$\int_{dt} \frac{3}{\sin^4 t \cos^2 t} = -\frac{1}{\sin^3 t \cos t} + 4\frac{1}{\sin t \cos t} - 8\cot t$$

$$\int_{dt} \frac{1}{\sin^3 t \cos^3 t} = \frac{1}{2}\frac{1}{\sin^2 t \cos^2 t} - \frac{1}{\sin^2 t} + 2\log(\tan t)$$

$$\int_{dt} \frac{1}{2+\sin t} = \frac{2}{\sqrt{3}}\arctan\left((1+2\tan(t/2))/\sqrt{3}\right)$$

$$\int_{dt} \frac{\sqrt{2}}{\sin t + \cos t} = \log\left(\frac{\tan(t/2) - 1 + \sqrt{2}}{\tan(t/2) - 1 - \sqrt{2}}\right)$$

$$\int_{dt} \frac{1}{1 + \tan t}$$

$$\int_{dt} \frac{1}{\sin(2t) + \cos t} = \frac{2}{3}\log\left(4\tan(t/2) + \tan^2(t/2) + 1\right) - \frac{1}{3}\log(\tan(t/2) - 1) - \log(\tan(t/2) + 1)$$

$$\int_{dt} \frac{2 + \sin t}{5 + \cos t} = \log\left(\tan^2(t/2) + 1\right) - \log\left(\tan^2(t/2) + 3\right) + \frac{\sqrt{6}}{3}\arctan\left(\sqrt{6}\tan(t/2)/3\right)$$

$$2\int_{dt} \frac{1 + \sin t}{1 + \sin t + \cos t} = t + \log\left(\tan^2(t/2) + 1\right)$$

$$\int_{dt} \frac{\sqrt{3}}{\sin^4 t + \cos^4 t} = \arctan\left((2\tan t - 1)/\sqrt{3}\right) + \arctan\left((2\tan t + 1)/\sqrt{3}\right)$$

$$\int_{dt} \frac{1}{1 + \sin t \cos t} = \frac{2}{\sqrt{3}}\arctan\left((2\tan t + 1)/\sqrt{3}\right)$$

$$2\int_{dt} \frac{\sin^2 t - \cos^2 t}{(\sin t + \cos t)^2} = -\log(1 + \sin(2t))$$

$$\int_{dt} \frac{1}{1 - \sin^4(2t)}$$

$$\int_{dt} \frac{6}{\cos^2(3t) - 1} = -\tan(3t/2) + {}^{3t/2}\tan^{-1}$$

$$\int_{dt} \frac{2\sqrt{6}}{2\cos^2 t - 3\sin^2 t} = \log\left(\frac{\sqrt{2} + \sqrt{3}\tan t}{\sqrt{2} - \sqrt{3}\tan t}\right)$$

$$\int_{dt} \frac{1}{\cos^3 t - 1}$$

inv trig integrals

$$\int_{dy} \arcsin y = y \arcsin y + \sqrt{1-y^2}$$

$$\int_{dy} y^2 \arcsin y = \frac{1}{3} y^3 \arcsin y + \frac{1}{9} \left(y^2+2\right) \sqrt{1-y^2}$$

$$\int_{dy} y^2 \arctan y = \frac{1}{3} y^3 \arctan y - \frac{1}{6} y^2 + \frac{1}{6} \log \left(y^2+1\right)$$

$$\int_{dy} y \arctan y = \frac{1}{2} y^2 \arctan y - \frac{1}{2} y + \frac{1}{2} \arctan y$$

$$\int_{dy} \frac{1}{\sqrt{1-y^2} \arcsin y} = \log (\arcsin y)$$

$$\int_{dy} \frac{\arcsin y}{y^2} = -\frac{\arcsin y}{y} - \log \left(\frac{1+\sqrt{1-y^2}}{y} \right)$$

$$\int_{dy} \frac{y \arcsin y}{\sqrt{1-y^2}} = \arcsin y \sqrt{1-y^2} - y$$

$$\int_{dy} \sqrt{1-y^2} \arcsin y = \frac{1}{2} y \sqrt{1-y^2} \arcsin y + \frac{1}{4} \arcsin^2 y - \frac{1}{4} y^2$$

$$\int_{dy} y \arccos y = \frac{1}{2} y^2 \arccos y - \frac{1}{4} y \sqrt{1-y^2} + \frac{1}{4} \arcsin y$$

$$\int_{dy} {}^{y}\overset{-1}{\cot}{}^{3}$$

$$\int_{dy} \overset{-1}{\cot} y = y \arctan y + \frac{1}{2} \log \left(1+y^2\right)$$

$$\int_{dy} \arctan (\sqrt{y})$$

$$\int\limits_{dy} y \arctan^2 y$$

$$\int\limits_{dy} \frac{y \arctan y}{\sqrt{y^2 + 1}}$$